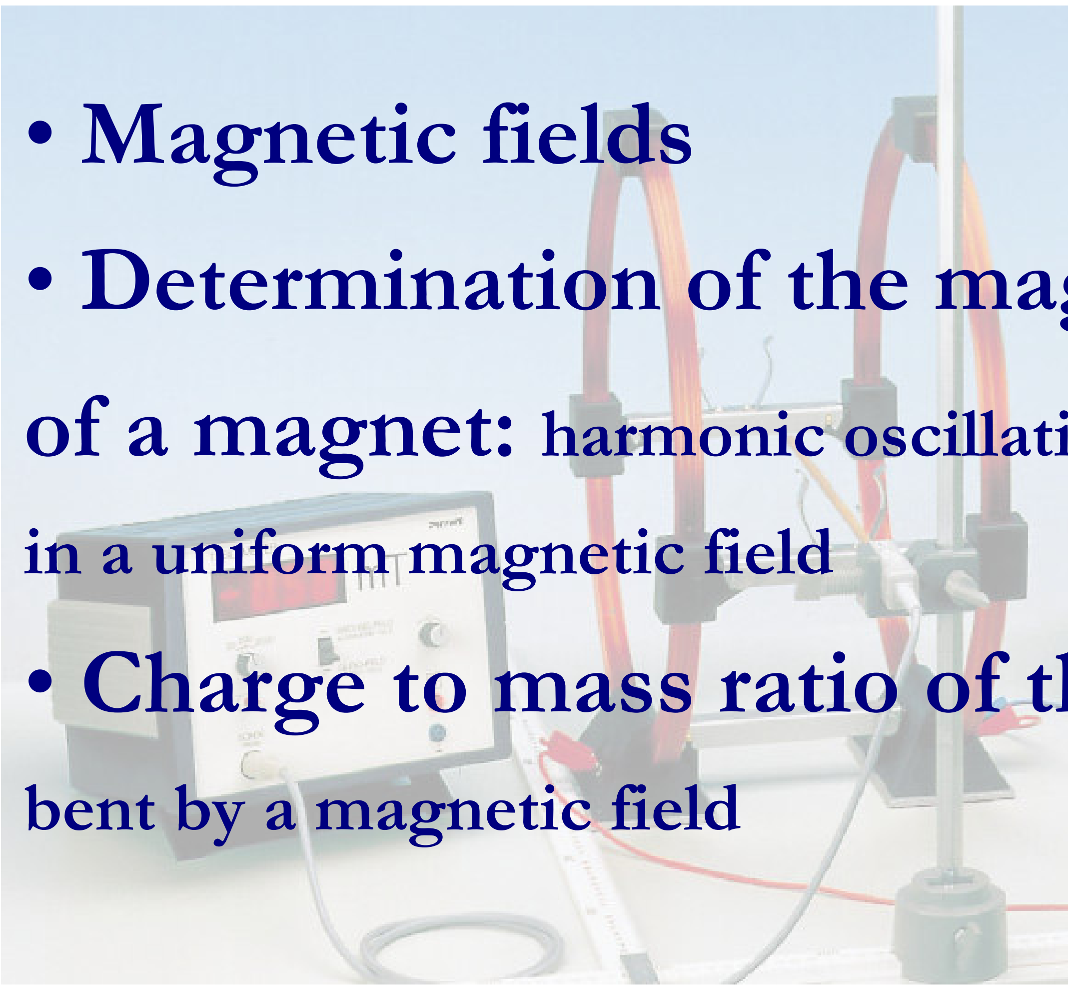
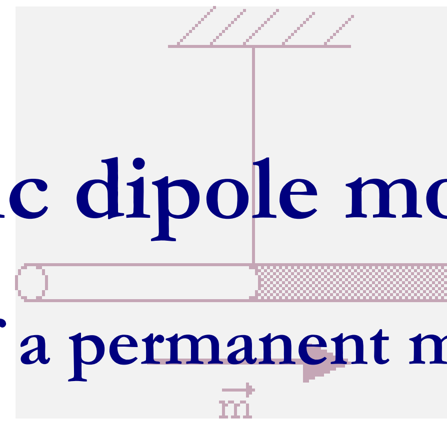
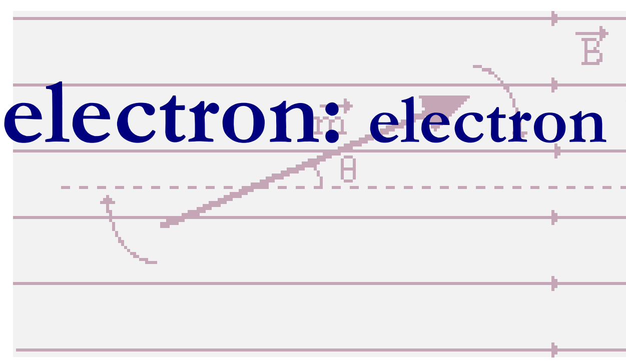
Experiment # 3: Determination of the magnetic moment of a magnet. Charge to mass ratio of the electron



- **Magnetic fields**
- **Determination of the magnetic dipole moment of a magnet:** harmonic oscillations of a permanent magnet in a uniform magnetic field
- **Charge to mass ratio of the electron:** electron beam bent by a magnetic field

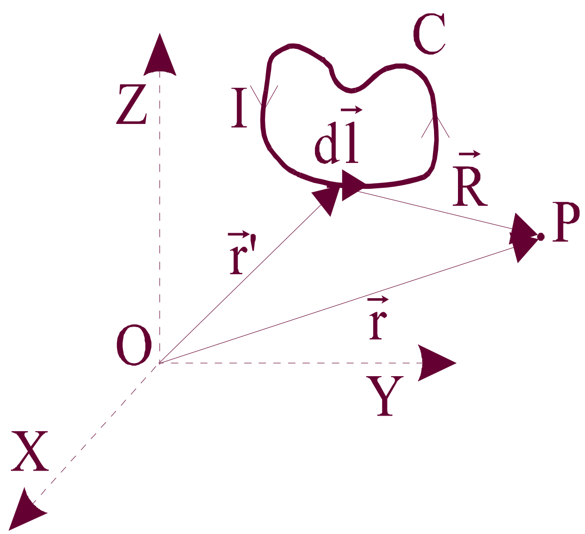




Magnetic fields (I)

Sources of magnetic fields: **moving charges or currents**

Units of magnetic field: SI, T (Tesla), Gauss is also used (1 T = 10^4 G)



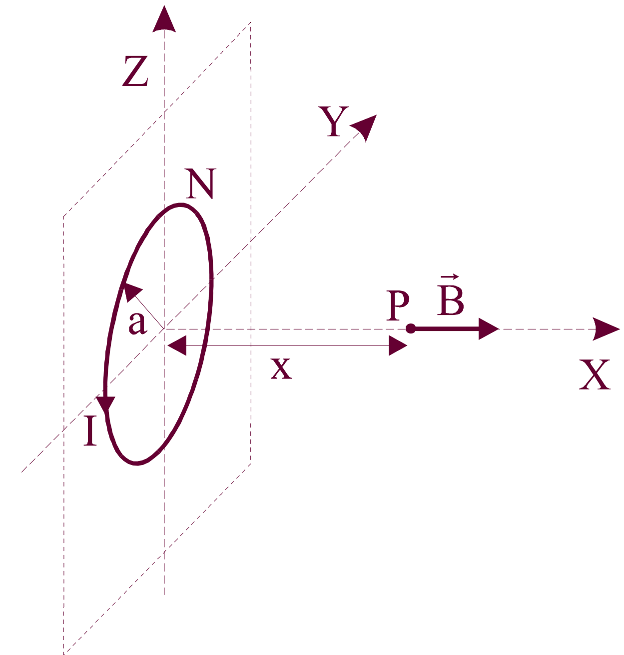
Biot-Savart law: field for a wire carrying a current *i*

$$\vec{B} = \frac{\mu_o}{4\pi} \oint_C \frac{i\left(d\vec{l} \times \vec{R}\right)}{R^3}$$

μ_0, permeability of free space

$$\mu_0 = 4\pi\ 10^{-7}\ N/A^2$$

Magnetic fields (II)



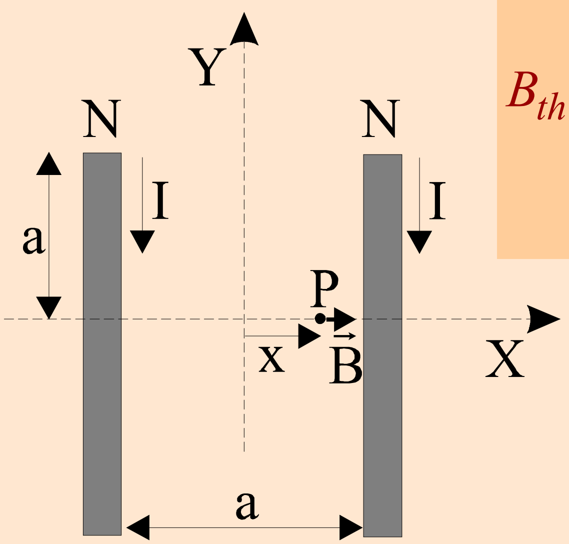
Magnetic field for a circular loop:

$$B_{th} = \frac{\mu_o}{2}\frac{a^2\, i}{\left(x^2 + a^2\right)^{3/2}}$$

Magnetic field for N loops:

$$B_{th} = \frac{\mu_o}{2}\frac{N\, a^2\, i}{\left(x^2 + a^2\right)^{3/2}}$$

Helmholtz coils (the distance between the two coils is equal to the radius of the coils)

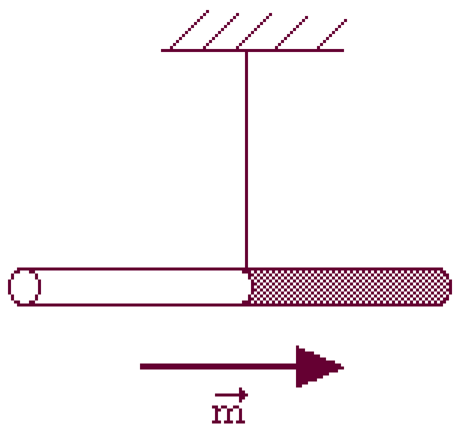


$$B_{th} = \frac{\mu_o N i a^2}{2}\left\{\frac{1}{\left[\left(x - a/2\right)^2 + a^2\right]^{3/2}} + \frac{1}{\left[\left(x + a/2\right)^2 + a^2\right]^{3/2}}\right\}$$

x = 0

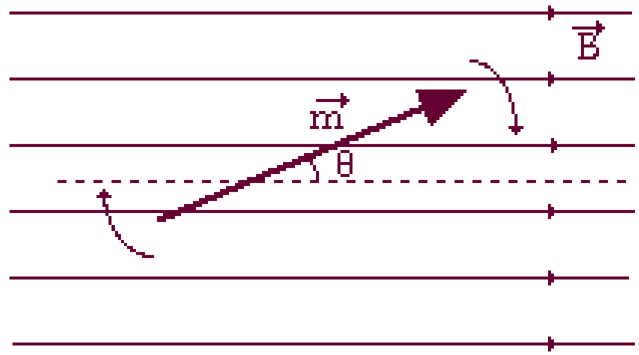
$$B_{th} = \frac{\mu_o N\, i}{a}\frac{8}{5^{3/2}}$$

Magnetic moment of a magnet (I)



$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\tau = \mu B \sin\theta$$



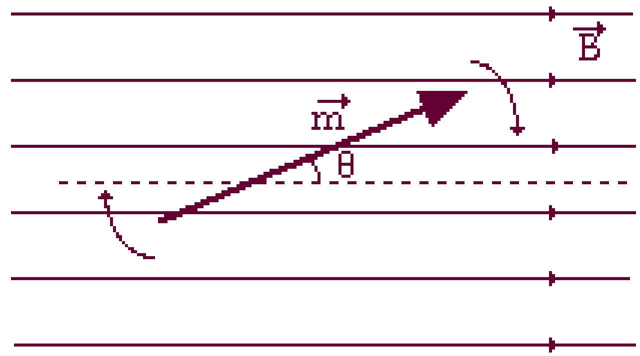
$$I \frac{d^2\theta}{dt^2} = \tau$$

small oscillations

$$I \frac{d^2\theta}{dt^2} + \mu B \theta = 0$$

$$\omega = \sqrt{\frac{\mu B}{I}}$$

Magnetic moment of a magnet (II)



$$T = \frac{2\pi}{\omega} = 2\pi\left(\frac{I}{\mu B}\right)^{1/2} \longrightarrow T^2 = \frac{5^{3/2}(2\pi)^2}{8\mu_0 N}\frac{I\,a}{\mu}\frac{1}{i}$$

$$B = \frac{\mu_o N i}{a}\frac{8}{5^{3/2}}$$

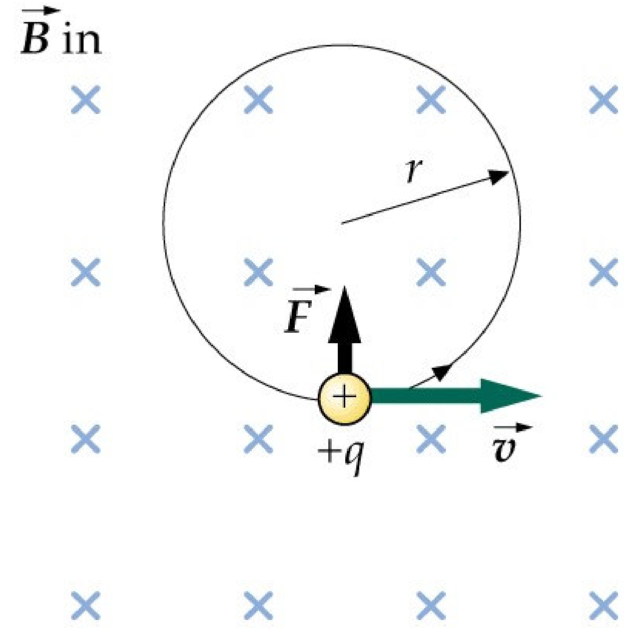
$$T^2 = cons.\frac{I\,a}{\mu}\frac{1}{i}$$

y (= T^2)   slope (= $cons.\frac{I\,a}{\mu}$) x (= $\frac{1}{i}$)

$$I = \frac{1}{12}M\left(3r^2 + L^2\right)$$

Charge to mass ratio of the electron

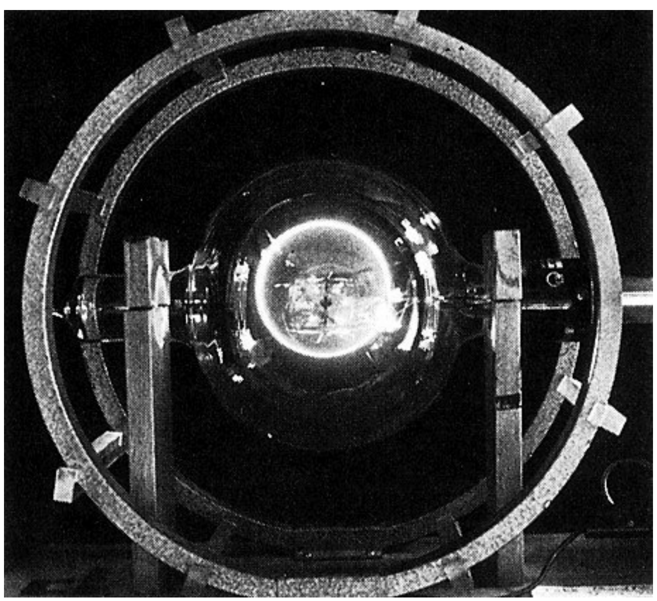
$$\vec{F}_e = q\vec{E}$$



$$\vec{F}_m = q\left(\vec{v} \times \vec{B}\right)$$

$$\frac{mv^2}{r} = evB$$

$$v = \frac{e}{m} rB$$

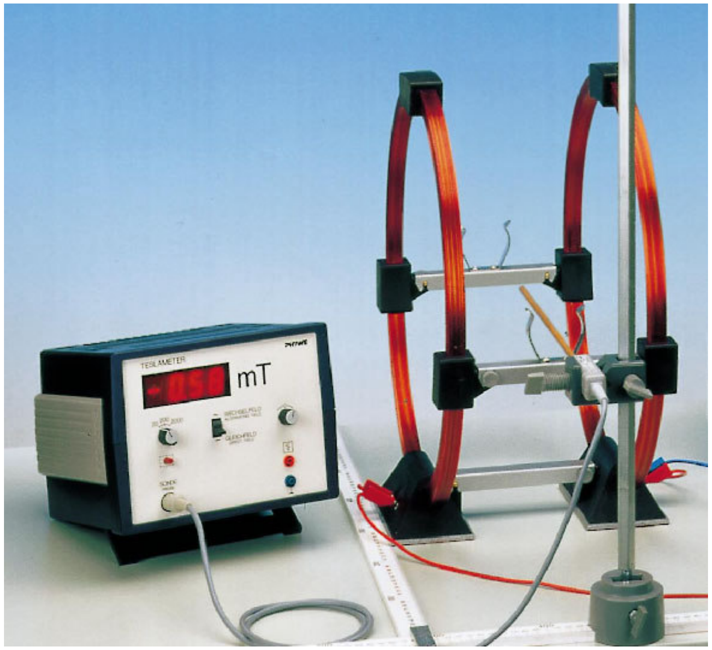


$$\frac{1}{2} mv^2 = eU$$

$$v = \left(2e\frac{U}{m}\right)^{1/2}$$

$$\frac{e}{m} = 2\frac{U}{r^2 B^2}$$

Equipment



MAGNETIC MOMENT of a MAGNET

- **Helmholtz coils**
- **(Teslameter)** HANDLE WITH CARE!
- **Mulimeter** (Ammeter, range A)
- **Power supply**
- **Cylindrical magnet** HANDLE WITH EXTREME CARE!

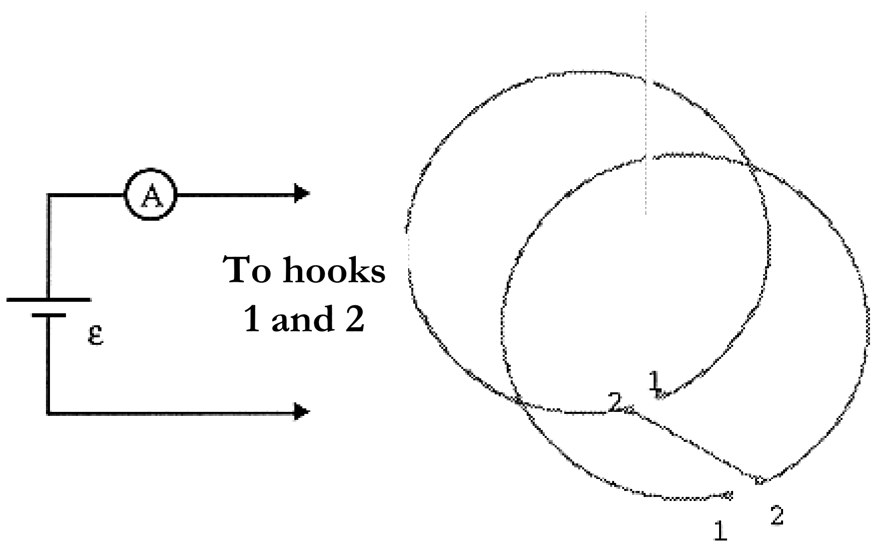
(check dimensions!!!)

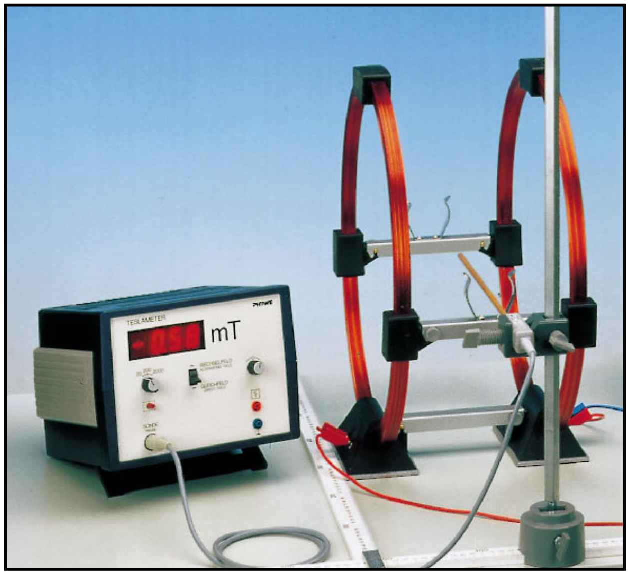
- **Stopwatch**
- **Connecting cords**
- **Support base and rods, and thread**

Experimental procedure (I)

Measuring magnetic fields: Use a teslameter to check the value and uniformity of the magnetic field in the central region of Helmholtz coils.

1. Teslameter calibration
2. Circuit set-up with Helmholtz coils and ammeter
3. Measurement of the magnetic field in the centre of the coils (average of the measurements of the two sides)





Experimental procedure (II)

Determination of the magnetic moment of a permanent magnet:

1. Hang a magnet on a thread tied to its central point, so that it oscillates in a horizontal plan, in the central region of the Helmholtz coils.
2. Calculate the moment of inertia of the magnet and its uncertainty: M = (77.60 ± 0.10) 10^{-3} Kg, r = (5.00 ± 0.05) 10^{-3} m, i L = (13.90 ± 0.10) 10^{-2} **m**
3. Make the magnet oscillate: shift it from its equilibrium position, and measure the period of oscillations for different currents (period of 20 oscillations)

i (A)	*t* (s)	T(s)	1/i (A^{-1})
0.75	–	–	–
1	–	–	–
1.25	–	–	–
...	–	–	–
...	–	–	–

μ is obtained from a least-squares fit

$$T^2 = \frac{5^{3/2}(2\pi)^2}{8\mu_0 N}\frac{I\ a}{\mu}\frac{1}{i}$$

Results

1. Test of the uniformity of a magnetic field in the central region of Helmholtz coils.
2. Determination of the moment of inertia of the magnet and its uncertainty.
3. Record of the oscillation periods for different currents, least-squares fit, and calculation of the magnetic moment of the magnet and its uncertainty.
4. Charge to mass ratio of the electron: Measurement of intensity (and magnetic field created), and of accelerating voltage. Calculation of the charge to mass ratio and contrast with data in literature.