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Self-adapting network topologies in congested scenarios

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Most studies in complex networks assume that once a link is created between two nodes it is never deleted. However, there is a recent interest towards systems where links can be rapidly rewired. An important issue in that type of networks is to discover the topology that, given a search algorithm, optimizes the search process. In this paper, we present a system model that, depending on the current network congestion, makes nodes to establish link connections so that the resulting topologies tend to a starlike when congestion is small and to randomlike topologies when congestion becomes relevant. Those topologies have been shown to be optimal in the above-mentioned conditions. Such a model can be easily implemented in practice and therefore, may be relevant in areas as the topology management of peer-to-peer networks.

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INTRODUCTION

In the last few years, there has been a great interest in understanding the topological properties of complex networks [1,2]. That interest comes in part from a need to understand the behavior of systems such as the Internet [3] and the World Wide Web [4].

Whereas most of those studies assume that once a link is created between two nodes, it is rarely deleted [5,6], our work is centered around dynamic communication networks where links can be rapidly rewired. This is motivated from the recent interest towards peer-to-peer networks [7–9].

An important issue is to discover the topology that, given a search algorithm, *optimizes* the search process (optimality is defined as the minimization of the average time to perform a search). Clearly, being able to obtain such topology structures seems to be a useful guide to drive the evolution of dynamic communication networks.

In [10], Guimerà *et al.* reported some interesting results regarding the structure of several topologies optimizing the search process in the presence of network congestion. First, they showed that if the number of parallel searches is small, such an optimal topology is a highly polarized starlike structure (a starlike topology, for k links per node, is formed by k central nodes with the rest of nodes connecting their outgoing links to these). However, this structure is inefficient if congestion considerations become relevant, since the central node may become overburdened. In fact, when nodes may get easily collapsed because of the packets they must deliver, the optimal network topology is a homogeneous-isotropic one. Furthermore and contrary to what one could expect, they also reported (by means of an optimization process carried out by using simulated annealing [11]) that the optimal topologies, instead of covering a wide range of structures, can be split in only these two categories: starlike topologies for a small number of parallel searches and homogeneous-isotropic topologies for large number of parallel searches, with a sharp transition between them.

In this paper, our goal is to provide a system model that, depending on the current network congestion, makes nodes to establish concrete link connections so that the resulting topologies perform not worse than both a starlike and a random network. From the previously cited results in [10], those topologies can be considered as optimal.

The mechanism we use for the node's election consists of assigning to each node a probability and making the election in accordance to it. More concretely, the *attachment kernel* Π_i that we use for such a task and that denotes the probability of being connected and/or rewired to node i has the form

$$\Pi_i \propto k_i^{\gamma_i}, \tag{1}$$

where k_i denotes the number of links of node i and

$$\gamma_i = \begin{cases} 2 & \text{if } c_i < \text{threshold} \\ 0 & \text{otherwise.} \end{cases}$$

The parameter c_i denotes the *traffic* (in packets per unit of time) that supports node i. We assume that the nodes have queues with the capacity to store as many packets as needed. This means that no packet is ever dropped. However, and without loss of generality, the processing power of a node is fixed to only one packet per unit of time. Hence, we say that a node *collapses* when $c_i \geqslant 1$.

The rationale behind the form of γ_i is explained as follows. First of all, we note that it is known [5] that by taking a value of $\gamma_i = 0$ (for all nodes) in Eq. (1) we obtain a random topology; in turn, if the value of γ_i is greater than 1 (e.g., 2), we obtain a starlike one [5]. Consequently, we establish that the value of γ_i will be either 2 if the traffic that supports node i is below a given threshold value, threshold, and 0 otherwise. That is, the network will evolve towards a randomlike topology when many of the nodes forming the network support a traffic above threshold and towards a starlike-like topology otherwise. The range of threshold will be between 0 (an empty node) and 1 (an almost-collapsed node).

In order to specify the congestion of a node i, we consider that each node generates packets at a rate ρ per unit of time, independently of the rest of the nodes. The destination of each of those packets is randomly fixed at the moment of its creation and they move in parallel according to a local search algorithm using minimum paths between nodes. For such a

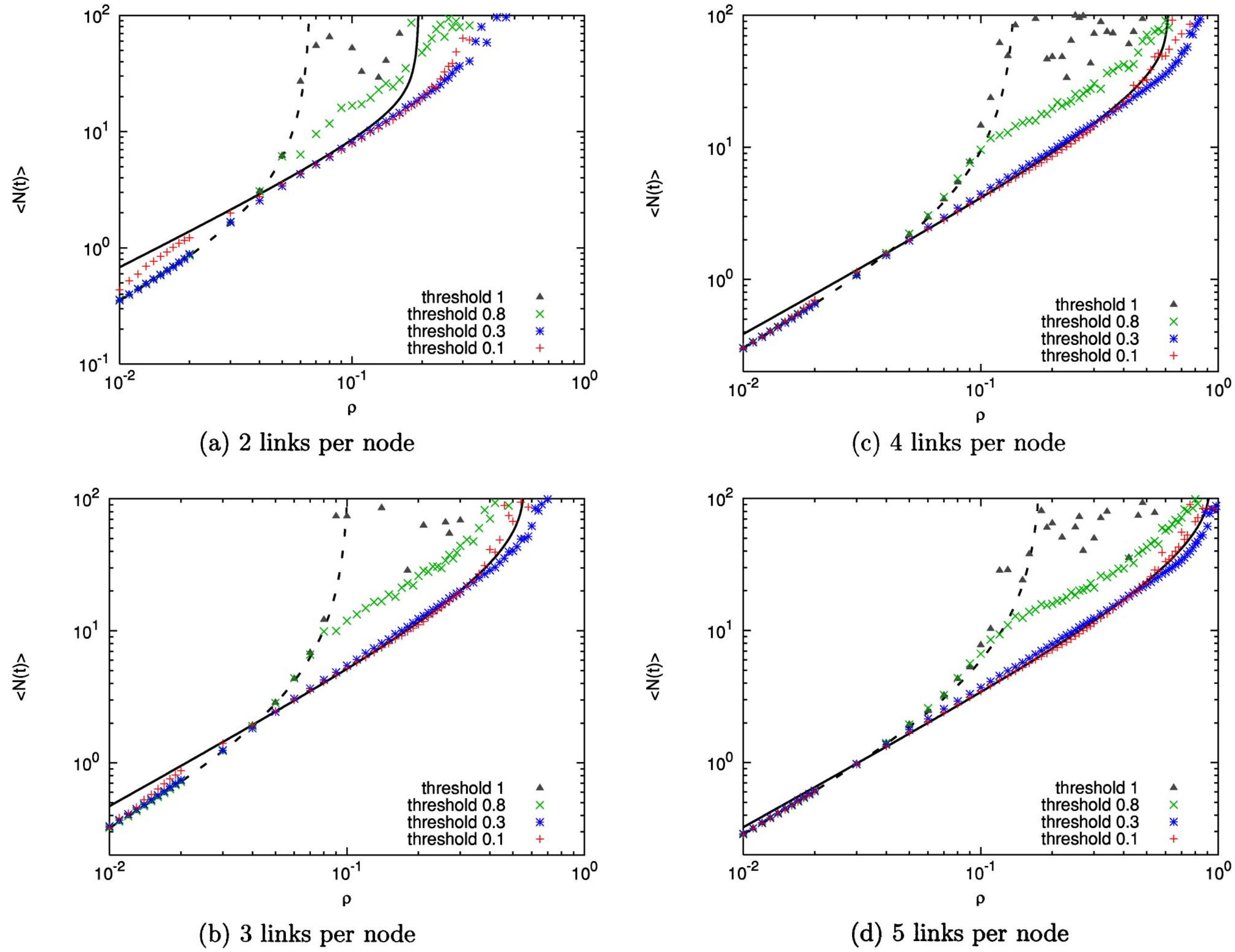

FIG. 1. (Color online) Average load ($\langle N(t)\rangle$) of several networks as a function of ρ for several values of the threshold (in log-log). We consider a scenario with 64 nodes and 2, 3, 4, and 5 links per node. The dashed and continuous lines respectively represent the average load of starlike and random networks.

model, we use the result of Guimerà *et al.* [10], which provides the number of packets that arrive at the node, on average, as a function of ρ, its betweenness centrality [12], denoted β_i, (the *betweenness centrality* of a node is defined as the number of routes, through shortest paths, that cross it) and the number of nodes in the system (denoted n),

$$c_i = \frac{\rho\beta_i}{n-1}. \qquad (2)$$

In order to evaluate how "good" a network topology is, we consider a scenario similar to [10]. Thus, we assume that the arrival of packets at a given node i is a Poisson process with mean c_i and that the delivery of packets is also a Poisson distribution. In that case, we have that the average size of the queues is given by [13]

$$\langle \nu_i\rangle = \frac{c_i}{1-c_i} = \frac{\dfrac{\rho\beta_i}{n-1}}{1-\dfrac{\rho\beta_i}{n-1}}, \qquad (3)$$

and the average load of the network $\langle N(t)\rangle$ is given by

$$\langle N(t)\rangle = \sum_{i=1}^{n}\langle \nu_i\rangle = \sum_{i=1}^{n}\frac{c_i}{1-c_i} = \sum_{i=1}^{n}\frac{\dfrac{\rho\beta_i}{n-1}}{1-\dfrac{\rho\beta_i}{n-1}}. \qquad (4)$$

According to Little's law [13], the average time needed by a packet to reach its destination is proportional to the total load of the network, and therefore minimizing $\langle N(t)\rangle$ is equivalent to minimizing the average cost of a search. Consequently, we consider a topology to be *optimal* if it minimizes the value of $\langle N(t)\rangle$.

At this point, we would like to remark that Eq. (2) is only valid for values of ρ such that c_i does not become collapsed. When at least one of the nodes in the network collapses (which can occur for values of ρ lower than 1), the average load of the network $\langle N(t)\rangle$ diverges [10].

TOPOLOGY EVOLUTION

Our experiments are carried out using simulations. At each simulation, we start with a starlike topology and in-

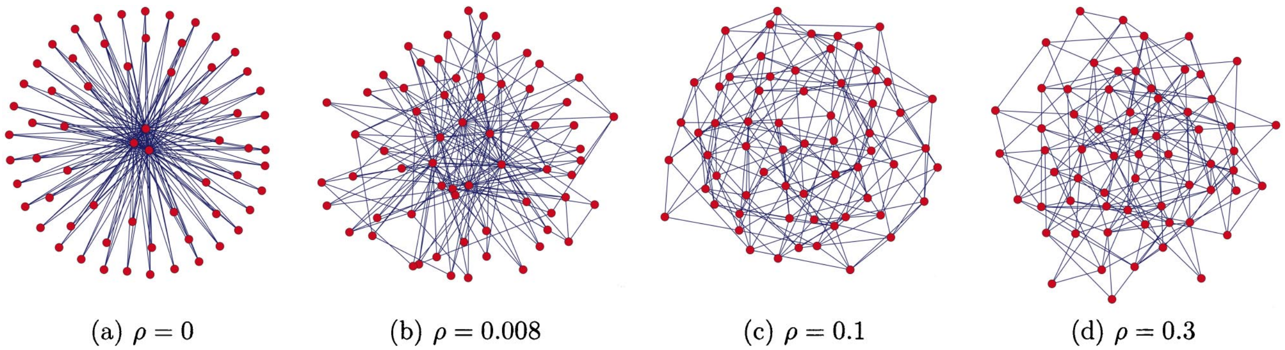

FIG. 2. (Color online) Illustration of several optimal topologies for different values of ρ in a scenario with 64 nodes, 3 links per node, and a threshold of 0.3.

crease the value of ρ (initially 0) until reaching the value such that some node becomes collapsed (i.e., until $c_i \geqslant 1$ for some i); at that point, we repeat the same process in inverse order.

For each value of ρ, we take a single node and perform one rewiring per link, using as attachment kernel the function in Eq. (1). Then, we repeat the same process for another node, until covering the whole set of nodes in the network (a *round*). We repeat that process five times and results are evaluated, for each value of ρ, at the end of the last round. We provide average values after repeating the experiments ten times. We have observed that the topology does not depend on whether we are increasing or decreasing the value ρ, but only on the value itself. Also, no disconnections have been observed.

We remark that the simulations are based on calculating the betweenness centrality and then obtaining the value of the congestion of each node by using Eq. (2), rather than making nodes to inject packets into the network and measuring the congestion of each node.

Performance evaluation. In our first experiment we take 64 nodes and vary the number of links that each node established with other nodes from 2 to 5. We also take as values of the threshold 0.1, 0.3, 0.8, and 1.

Figure 1 shows the result of evaluating the value of $\langle N(t) \rangle$ for those scenarios (similar results where obtained by using 32 and 128 nodes). We found that for small values of ρ, starlike topologies outperform random topologies, and for high values of ρ random topologies outperform starlike topologies (although in some cases the difference was minimal). This is consistent with previous results [10].

Depending on the threshold value, curves have a different behavior. For moderate and high values of ρ, the curves with high thresholds (i.e., 0.8 and 1) are the most loaded. At the other extreme, for small values of ρ, the most loaded curve has threshold of 0.1 (this is more evident in the case where there are two links). In between (i.e., with 0.3), we found that the load of the corresponding curve follows the dashed curve until it crosses the continuous curve, and then follows this one (see Fig. 1). Therefore, and taking into account the results in [10], it can be argued that our topologies with a threshold of 0.3 are to optimal. Intuitively, this can be explained if we take into account the fact that nodes self-adapt dynamically (by means of modifying the probability of being connected) to avoid congestion, which results in a minimization of the load of the network. Also, it allows us to explain why, for high values of ρ, they perform even better than randomlike networks: whereas random networks uniformly distribute the load among all nodes, statistically, there is always a node more connected than the rest (i.e., with a bigger betweenness centrality). Such nodes, for high values of ρ, are the first to become overloaded, thus quickly increasing the load of the network. On the contrary, the use of thresholds permits us to minimize the number of overloaded nodes, minimizing the load of the network. In Fig. 2, we illustrate the form of the obtained topologies.

To analyze the topology evolution, we focus on the cumulative degree distribution $P(k)$ when we vary the injection rate ρ. Such a metric represents the probability that a randomly selected node has exactly k or more edges. We consider the case where there are three links per node and threshold of 0.3. The results of our simulations are shown in Fig. 3. For $\rho=0$ we have that $\gamma_i=2$ for all nodes and consequently $\Pi_i \propto k_i^2$; therefore, we have a starlike topology. At the other extreme, for $\rho=1$ we have that $\gamma_i=0$ for many nodes

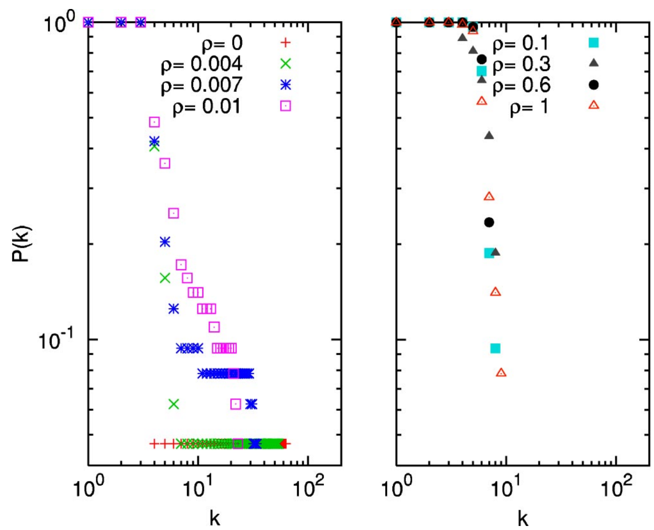

FIG. 3. (Color online) Cumulative degree distribution for several values of ρ (in log-log) in a scenario with 64 nodes, 3 links per node, and a threshold of 0.3. The horizontal axis is the vertex degree and the vertical axis is the probability distribution of degrees (i.e., the fraction of vertices that have a degree equal to k or higher).

and consequently Π_i will be the same for all of them; thus, the network will tend to be random. In between, there is a transition from one type of topology to the other. This can be observed by looking at the form of curves: for values of ρ above 0.05, curves appear to have an exponential distribution (typical of random networks) and below it, curves quickly evolve towards a starlike cumulative degree distribution curve.

Deployment of the model. In order to justify that the proposed model can be easily implemented in practice, we first point out the fact that our approach consists of making the nodes establish connections with other nodes by considering only their vertex degree (which can be easily obtained) and their congestion level. Despite that, to obtain a node's congestion we used the betweenness centrality (which can only be obtained by means of having global network information); in a realistic case, congestion can be directly measured from the node's local state.

Furthermore and for the sake of simplicity, to perform the node's election we considered the whole set of nodes, although it seems feasible to consider only a subset of them. As a matter of fact, Newman's result [14] shows that our approach is also valid if we consider, as a potential node's target, those reached along a random walk.

On the other hand, an important aspect of our model is the *convergence period* between topologies. We define it as the number of rounds that must pass between two "stable" topologies with different values of ρ. Our experiments have shown that it is very small, both when we increase the injection rate ($\approx$1 round) and when we decrease it ($\approx$5 rounds), which shows that our model reacts very fast to any change in the injection rate.

In summary, our work presents a dynamical model for topology adaptation that takes into account the congestion level at the nodes. As it has been shown, the resulting network topologies optimize the search process in scenarios where links can be easily rewired, such as the topology management of peer-to-peer networks [7–9]. Furthermore, such a model can be easily deployed in practice.

Some issues are still open for future work. Currently, we are studying the case when there are restrictions on how nodes can establish links (e.g., scenarios where nodes may be physically separated at a distance that overpasses their coverage distance).

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