Bounds on Stability and Latency in Wireless Communication

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***Abstract*—In this paper, we study stability and latency of routing in wireless networks where it is assumed that no collision will occur. Our approach is inspired by the adversarial queuing theory, which is amended in order to model wireless communication. More precisely, there is an adversary that specifies transmission rates of wireless links and injects data in such a way that an average number of data injected in a single round and routed through a single wireless link is at most r, for a given $r \in (0, 1)$. We also assume that the additional "burst" of data injected during any time interval and scheduled via a single link is bounded by a given parameter b.**

Under this scenario, we show that the nodes following so called *work-conserving* scheduling policies, not necessarily the same, are guaranteed stability (i.e., bounded queues) and reasonably small data latency (i.e., bounded time on data delivery), for injection rates $r < 1/d$, where d is the maximum length of a routing path. Furthermore, we also show that such a bound is asymptotically optimal on d.

***Index Terms*—Network stability, latency, wireless networks, adversarial queuing theory.**

I. Introduction

In this paper, we consider a multihop wireless network where data is transmitted from its source node to its destination node through other intermediate nodes.

One crucial issue to characterize the performance of a networks is that of *stability*. Roughly speaking, a communication network system is said to be stable if data waiting to be delivered (backlog) is finitely bounded at any single time. The importance of such an issue is obvious, since if one cannot guarantee stability, then one cannot hope for ensuring deterministic guarantees for most of the network performance metrics. One such metric is *latency*, defined as the maximum time for delivering data from its source to its destination, taken over all data occurring in the routing process.

Whereas in the last few years much of the analysis of worst-case behavior of multihop wireline networks and scheduling policies has been performed using *adversarial* models, which try to create as much trouble for the scheduling algorithm as possible [1], [2], only a few papers have been focussed on wireless networks. In [3], Borodin *et al*. considered a model in which each node can transmit, at each time step, to all its neighbors, and show that the Nearest-to-Go scheduling policy is stable. They also showed that the Longest-in-System policy is unstable. Andrews *et al*. [4], in a model in which a node can transmit to only one neighbor at a time step, provided some fully distributed scheduling algorithms that ensure network stability, both when the routes are specified by the adversary and when they are chosen by the nodes.

Contrary to the previous papers, which assumed that data doesn't suffer collisions when several nodes transmit at the same time, in [5] Chlebus *et al*. studied stability of some distributed broadcast protocols. However, they assumed a scenario in which the transmission range of each node reaches all the other nodes. The maximum throughput, defined to mean the maximum rate for which stability is achievable, was studied by Chlebus *et al*. [6]. Anantharamu *et al*. [7] extended this work by studying the impact of limiting the adversary by assigning independent rates of injecting data to each node.

In this paper, we study stability in a scenario formed by a multihop wireless network, where each node has a, possibly different, work-conserving scheduling policy. We say a scheduling policy is *work-conserving* if it cannot be idle as long as there is data queued to be transmitted. Many well-known scheduling policies like FIFO (First-In-First-Out), LIS (Longest-In-System), SIS (Shortest-In System), FTG (Farthest-To-Go), NTS (Nearest-To-Source), etc., are work-conserving policies, whereas other policies like Round-Robin, GPS (Generalized Processor Sharing), WFQ (Weighted Fair Queueing), etc., are non-work-conserving.

Our main result shows that a network with nodes following a work-conserving scheduling policy is stable provided the data injection rate is lower than $1/d$, being d the largest number of links that data can cross in the network. Furthermore, we also show that such a bound is asymptotically optimal on d.

The rest of the paper is organized as follows. In Section II we introduce our adversarial model and in Section III we present the main results about stability and latency of wireless communication in the specified model.

II. The model

We use a modified version of the wireless adversarial model proposed by Andrews *et al*. [4]. We consider a wireless multihop undirected network of n nodes, where each node acts as both a transmitter and a receiver. When data is transmitted from its source node to its destination node and they are too far away from each other to communicate, data may go through other nodes as intermediate hops. Each node contains a queue for each outgoing link and uses it to store there data to be sent along the corresponding link. We assume that data is fluid-like (in the sense that the unit to transmit can be as small as needed), and that several pieces of data may be transmitted

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along one link in one time step. Furthermore, we assume that data units don't suffer collisions.[1] This feature is similar to the wireline adversarial model, that also doesn't take into account collisions between packets.

Time is divided into fixed slots. Each node can transmit at different capacities in the interval $[0, 1]$, which may or may not vary over time as a result of changing wireless channel conditions. We use $r_{ij}(t)$ to denote the rate at which node i can transmit to node j at time slot t, also referred as *transmission rate*. It is assumed that the transmission rate is defined over all pair of nodes, since $r_{ij}(t)$ can be set to zero if nodes i and j are too far away from each other to communicate directly. Furthermore, we assume that a node can transmit to only one neighbor at each time step.

The time evolution is seen as a game between a *scheduling queue policy* which decides, at each time step, which data must be transmitted (if any), and a bounded *adversary* that governs both the *data arrivals* and the *channel conditions*, i.e., the transmission rates.

The adversary. Regarding the data arrivals, at each time step the adversary injects a set of data into some of the nodes in the network. More precisely, such an injection is defined by a pair of parameters (b, r), where $b \geq 1$ is a natural number and r satisfies $0 \leq r < 1$. The parameter b (usually called *burstiness*) models the short bursts of data the adversary can inject into the network. The parameter r (called the *injection rate*) models the long-term rate at which data can be injected into the network. The adversary is free to choose both the source and the destination node for any injected data. It also specifies the routing path from the source to the destination that data must follow. Paths don't include the same link more than once, and data is absorbed after traversing its route.

The adversary also controls the quality of channels between nodes, trying to create as much trouble for the scheduling policy as possible, by means of specifying the transmission rates. At each time slot and for each node i, the adversary sets up the values of the rate vector $(r_{i1}(t), r_{i2}(t), ..., r_{in}(t))$ before node i makes its scheduling decision. These rates are not know to the scheduling algorithm.

In order for stability to be feasible, it is necessary to impose some restrictions on the adversary so that it would not be able to fully load any link a priori. More specifically, we require that the adversary satisfies the following *admissibility condition*. Let $I_{ij}(t)$ represent the total amount of data that the adversary injects at time t and has link ij on its path. We say that the adversarial injection is *admissible for rate r and burst b* if there exist fractions $x_{ij}(t) \in [0, 1]$ such that

$$\sum_j x_{ij}(t) = 1, \quad \forall i,\ \forall t \tag{1}$$

$$\sum_{t \in T_x} I_{ij}(t) \leq r \sum_{t \in T_x} r_{ij}(t) x_{ij}(t) + b, \quad \forall ij,\ \forall T_x \tag{2}$$

where T_x denotes a consecutive sequence of x time steps. One can view $x_{ij}(t)$ as representing fractional decisions that indicate the assignment of data injected by the adversary that wishes to pass through node i at each time step. The admissibility condition of Eq. 1 (combined with Eq. 2) says that the total size of such a data is, on average, at most r.

Stability. In order to formally define stability, we denote by d_p the number of queues that a data unit p has to cross. Furthermore, we denote by a_i^p and f_i^p the time instants that p respectively arrives at and departs from the ith queue on its routing path, where $1 \leq i \leq d_p$. If p leaves its ith queue in time step f_i^p, it will arrive at its $(i+1)$st queue at time step $a_{i+1}^p = f_i^p$. Finally, we denote by Q_i^p the time p spends in the ith queue on its path, i.e., $Q_i^p = f_i^p - a_i^p$. Let $Q = \max_{p, 1 \leq i \leq d_p} Q_i^p$.

Given an adversary $\mathcal{A}$ (as defined above) and a scheduling protocol $\mathcal{P}$, we say a network $\mathcal{G}$ is *stable* if $Q \leq \infty$ [4].

III. Stability Conditions and Latency of Routing with Work-Conserving Scheduling Policies

In this section, we obtain a formula for the threshold value on data injection rate guaranteeing stability in wireless networks with work-conserving scheduling policies (i.e., nodes cannot be idle as long as there data queued to be transmitted). Furthermore, we also estimate data latency for injection rates below this threshold value.

We remark that each node may have its own, possibly different, scheduling policy (FIFO, LIFO, Longest-in-System, etc.), as long as they are work-conserving. Furthermore, the scheduling policies don't need to know the quality of the transmission channels (i.e., the values of the rate vectors), since they only take care of deciding the order in which data is transmitted.

The following theorem provides a bound on the injection rate that guarantees network stability under any work-conserving scheduling policies.

Theorem 1: Any network in which all queues use a, possibly different, work-conserving scheduling policy and data are injected by a (b, r)-adversary, is stable for $r < \frac{1}{d}$, where d is the largest number of hops that any data unit traverses in the network. Furthermore, data latency is bounded from above by $d\frac{b\Delta}{1-rd}$, where Δ denotes the maximum number of neighbors a node can have.

Proof: The proof has two parts. First, we show that if $r < \frac{1}{d}$ then the maximum time interval data takes to cross any queue is bounded, which implies stability. Second, we prove that data latency is also upper bounded by $d\frac{b\Delta}{1-rd}$, provided the first condition on stability $r < \frac{1}{d}$ holds.

In what follows, we denote as $N(i)$ the set of nodes that are neighbors of node i. We also note that $d = max_p\{d_p\}$.

Remark 1: Note that we don't assume, a priori, whether the scenario formed by the network, the scheduling policy, and the adversary, is stable or not. Thus, if it is unstable, the time p takes to leave its ith queue could be infinite (i.e., $f_i^p = \infty$).

Remark 2: Note that if $f_i^p = \infty$ (for some p) then $Q = \infty$. However, we base our proof of finding under which conditions, $Q < \infty$ (which will automatically imply $f_i^p < \infty$).

Part (1): Let p be a data unit that attains the maximum Q (i.e., $Q_i^p = Q$) at the ith queue on its path. We will call it the *ith queue of data p.*

[1]This can be achieved by making a specific channel assignment based on Time/Frequency/Code division or other methods for resolving contention in the data-link layer.

Let t_B be the oldest time step such that (1) $t_B < a_i^p$, and (2) in every step in $(t_B, a_i^p]$ the ith queue is non-empty. Hence, we have that during the interval $(t_B, f_i^p]$ the ith queue is non-empty.

Define ϕ_i^p as the set formed by all data units served by the ith queue during the interval $(t_B, f_i^p]$, and let p^* be the oldest data unit in ϕ_i^p (i.e., $\forall p' \in \phi_i^p$ $(a_1^{p'} \geq a_1^{p^*})$). Hence, by the definition of p^*, all data in ϕ_i^p must have been injected during the interval $[a_1^{p^*}, f_i^p]$.

Based on the above mentioned scenario and on the definition of the adversarial model, $Q_i^p = f_i^p - a_i^p$ is bounded by the maximum number of data units injected during the interval $[a_1^{p^*}, f_i^p - 1]$ (i.e., the worst-case scenario is: where all data injected since the time instant $a_1^{p^*}$ until p is served, cross the ith queue of p and is scheduled before p) minus the data served by the ith queue of p during the interval $[t_B, a_i^p]$. Recall that in each step in the period $[t_B, a_i^p]$ the ith queue of p is non-empty. We have

$$\begin{aligned} & f_i^p - a_i^p \\ & \leq \sum_{j \in N(i)} \left(r \sum_{t=a_1^{p^*}}^{f_i^p - 1} r_{ij}(t) x_{ij}(t) + b \right) - \sum_{j \in N(i)} \sum_{t=t_B}^{a_i^p} r_{ij}(t) x_{ij}(t) \\ & = \sum_{j \in N(i)} \left(r \sum_{t=a_1^{p^*}}^{t_B - 1} r_{ij}(t) x_{ij}(t) + r \sum_{t=t_B}^{a_i^p} r_{ij}(t) x_{ij}(t) \right. + \\ & \left. \quad r \sum_{t=a_i^p+1}^{f_i^p - 1} r_{ij}(t) x_{ij}(t) + b - \sum_{t=t_B}^{a_i^p} r_{ij}(t) x_{ij}(t) \right) \end{aligned}$$

Now, taking into account that $r \leq 1$, we have

$$\begin{aligned} & f_i^p - a_i^p \\ & \leq \sum_{j \in N(i)} \left(r \sum_{t=a_1^{p^*}}^{t_B - 1} r_{ij}(t) x_{ij}(t) + r \sum_{t=a_i^p+1}^{f_i^p - 1} r_{ij}(t) x_{ij}(t) + b \right) \end{aligned}$$

and taking also into account that $r_{ij}(t) \leq 1$, we finally obtain

$$f_i^p - a_i^p \quad \leq \sum_{j \in N(i)} \left(r \sum_{t=a_1^{p^*}}^{t_B - 1} x_{ij}(t) + r \sum_{t=a_i^p+1}^{f_i^p - 1} x_{ij}(t) + b \right)$$

Let k be the hop number of p^* when it arrives to the queue where p attains the maximum Q. Taking into account the first admissibility condition (Eq. (1)) we have that $\sum_{j \in N(i)} x_{ij}(t) = 1$ for all t, where $x_{ij}(t) \in [0, 1]$. Therefore,

$$\begin{aligned} & f_i^p - a_i^p \\ \leq\ & r(t_B - a_1^{p^*}) + r(f_i^p - a_i^p - 1) + |N(i)| \cdot b \\ =\ & r(t_B - a_k^{p^*} + a_k^{p^*} - a_1^{p^*}) + r(f_i^p - a_i^p - 1) + |N(i)| \cdot b \\ =\ & r(t_B - a_k^{p^*}) + r(a_k^{p^*} - a_1^{p^*}) + r(f_i^p - a_i^p - 1) + \\ & |N(i)| \cdot b \end{aligned}$$

Since $a_k^{p^*} \geq t_B$, then we have

$$f_i^p - a_i^p \quad \leq\ r(a_k^{p^*} - a_1^{p^*}) + r(f_i^p - a_i^p - 1) + |N(i)| \cdot b$$

Since $f_i^p - a_i^p = Q_i^p = Q$ and $a_k^{p^*} - a_1^{p^*} \leq (d-1)Q$, and taken into account that Q is the maximum time a data unit takes to cross a queue, and $d - 1$ is the maximum number of queues a data unit crosses until reaching its last queue, we have that

$$\begin{aligned} Q &\leq rQ(d-1) + r(Q-1) + |N(i)| \cdot b \\ Q &\leq rQd - r + |N(i)| \cdot b \end{aligned}$$

It follows that $Q < \infty$ for $r < 1/d$.

Part (2): Consider a data unit p that traverses a path with d_p queues, where $d_p \leq d$. This network satisfies the property delivered in Part (1). Call δ_p the latency of p and let $\Delta = \max_i |N(i)|$. From the above derivation we see that $\delta_p \leq d_p Q \leq d \frac{b\Delta - r}{1 - rd} \leq d \frac{b\Delta}{1 - rd}$.

■

Tightness of the bounds.

In [8], Charny and Le Boudec shown that, in the *wireline model* (here, we refer to the "standard" adversarial queueing model [1] for wireline networks), there exists a large enough network which considers constant rate links and the FIFO scheduling policy, that becomes unstable when $r > 1/(d-1)$. However, we can simulate the behavior of such a network in a wireless scenario by using the same iterative steps as in [8] (note that such simulation may not be possible in general, since the models are different); for details see [9].

This implies that any bound for stability must be lower or equal to $1/(d-1)$. Let's refer to this as the *optimistic* bound.

Now, we define a parameter $\epsilon(d)$ measuring the difference between such an optimistic stability bound (i.e., $1/(d-1)$) and the bound provided in Theorem 1 (i.e., $1/d$) when we increase d. We have that it behaves like $\epsilon(d) \sim \frac{1}{d-1} - \frac{1}{d} \sim \frac{1}{d^2}$. Therefore, we have that our bound in Theorem 1 is asymptotically optimal within that limit.

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