

## Table of Direct Products of Irreducible Representations

### General Rules

$$\begin{array}{lll}
 A \times A = A & B \times B = A^{\ddagger} & A \times B = B \\
 A \times E = E & A \times E_1 = E_1 & A \times E_2 = E_2 \\
 B \times E = E & B \times E_1 = E_2 & B \times E_2 = E_1 \\
 A \times T = T & B \times T = T & 
 \end{array}$$

### Superscripts and Subscripts

$$\begin{array}{lll}
 ' \times ' = ' & '' \times '' = ' & ' \times '' = '' \\
 g \times g = g_u \times u = g_u \times g = u & & 1 \times 1 = 1_1 \times 2 = 2_2 \times 2 = 1 \quad \S
 \end{array}$$

### Doubly degenerate representations<sup>¶</sup>

For  $C_3, C_{3h}, C_{3v}, D_3, D_{3h}, D_{3d}, C_6, C_{6h}, C_{6v}, D_6, D_{6h}, S_6, O, O_h, T, T_d, T_h$

$$E_1 \times E_1 = E_2 \times E_2 = A_1 + [A_2] + E_2 \quad E_1 \times E_2 = B_1 + [B_2] + E_1$$

For  $C_4, C_{4v}, C_{4h}, D_{2d}, D_4, D_{4h}, S_4$

$$E \times E = A_1 + [A_2] + B_1 + B_2$$

(If no subscripts on  $A, B,$  or  $E,$  read as  $A_1 = A_2 = A$  etc.)

### Triply degenerate representations<sup>¶</sup>

For  $T_d, O, O_h$

$$\begin{array}{l}
 E \times T_1 = E \times T_2 = T_1 + T_2 \\
 T_1 \times T_1 = T_2 \times T_2 = A_1 + E + [T_1] + T_2 \\
 T_1 \times T_2 = A_2 + E + T_1 + T_2
 \end{array}$$

### Linear molecules ( $C_{\infty v}, D_{\infty h}$ )<sup>¶</sup>

$$\begin{array}{ll}
 \Sigma^+ \times \Sigma^+ = \Sigma^- \times \Sigma^- = \Sigma^+ & \Sigma^+ \times \Sigma^- = \Sigma^- \\
 \Sigma^+ \times \Pi = \Sigma^- \times \Pi = \Pi & \Sigma^+ \times \Delta = \Sigma^- \times \Delta = \Delta \\
 \Pi \times \Pi = \Sigma^+ + [\Sigma^-] + \Delta & \Delta \times \Delta = \Sigma^+ + [\Sigma^-] + \Gamma \\
 \Pi \times \Delta = \Pi + \Phi & 
 \end{array}$$

<sup>‡</sup> Except in the groups  $D_2$  and  $D_{2h}$ :  $B_1 \times B_2 = B_3, B_2 \times B_3 = B_1, B_3 \times B_1 = B_2$ .

<sup>§</sup> Except in the groups  $D_2$  and  $D_{2h}$ :  $1 \times 2 = 3, 2 \times 3 = 1, 1 \times 3 = 2$ .

<sup>¶</sup> The direct product of a degenerate species with itself may be resolved into a symmetric direct product, and an anti-symmetric direct product. In vibrational spectroscopy, the symmetry species of the overtones of a degenerate fundamental are obtained from the symmetric direct products. In the determination of electronic terms, the symmetric and anti-symmetric direct products for orbital angular momentum are taken with the appropriate spin functions to ensure that the total wave functions are anti-symmetric. The anti-symmetric component of the direct products are placed within [ ] in the table above.