

TABLE I. Character tables of permutation groups.

\mathcal{P}_2	$\chi^2(\mathbf{R})$	$\chi(\mathbf{R}^2)$						
\mathcal{A}'	1	1						
\mathcal{A}''	1	-1						
\mathcal{P}_3	$\chi^3(\mathbf{R})$	$2\chi(\mathbf{R}^3)$	$3\chi(\mathbf{R})\chi(\mathbf{R}^2)$					
\mathcal{A}_1	1	1	1					
\mathcal{A}_2	1	1	-1					
\mathcal{E}	2	-1	0					
\mathcal{P}_4	$\chi^4(\mathbf{R})$	$8\chi(\mathbf{R})\chi(\mathbf{R}^3)$	$3\chi^2(\mathbf{R}^2)$	$6\chi(\mathbf{R}^4)$	$6\chi^2(\mathbf{R})\chi(\mathbf{R}^2)$			
\mathcal{A}_1	1	1	1	1	1			
\mathcal{A}_2	1	1	1	-1	-1			
\mathcal{E}	2	-1	2	0	0			
\mathcal{F}_1	3	0	-1	1	-1			
\mathcal{F}_2	3	0	-1	-1	1			
\mathcal{P}_5	$\chi^5(\mathbf{R})$	$15\chi(\mathbf{R})\chi^2(\mathbf{R}^2)$	$30\chi(\mathbf{R})\chi(\mathbf{R}^4)$	$20\chi^2(\mathbf{R})\chi(\mathbf{R}^3)$	$24\chi(\mathbf{R}^5)$	$20\chi(\mathbf{R}^2)\chi(\mathbf{R}^3)$	$10\chi^3(\mathbf{R})\chi(\mathbf{R}^2)$	
\mathcal{A}_1	1	1	1	1	1	1	1	
\mathcal{A}_2	1	1	-1	1	1	-1	-1	
\mathcal{G}_1	4	0	0	1	-1	-1	2	
\mathcal{G}_2	4	0	0	1	-1	1	-2	
\mathcal{H}_1	5	1	-1	-1	0	1	1	
\mathcal{H}_2	5	1	1	-1	0	-1	-1	
\mathcal{I}	6	-2	0	0	1	0	0	

TABLE II. The symmetrized cubes of the irreducible representations of non-centrosymmetric point groups.

C_3	$E^3 = \{2A + E \mathcal{A}_1\} + \{E \mathcal{E}\}$
C_{3h}	$E'^3 = \{2A' + E' \mathcal{A}_1\} + \{E' \mathcal{E}\}$ $E''^3 = \{2A'' + E'' \mathcal{A}_1\} + \{E'' \mathcal{E}_1\}$
C_{3v}, D_3	$E^3 = \{A_1 + A_2 + E \mathcal{A}_1\} + \{E \mathcal{E}\}$
D_{3h}	$E'^3 = \{A'_1 + A'_2 + E' \mathcal{A}_1\} + \{E' \mathcal{E}\}$ $E''^3 = \{A''_1 + A''_2 + E'' \mathcal{A}_1\} + \{E'' \mathcal{E}\}$
$C_4, D_{2d}, S_4,$ C_{4v}, D_4	$E^3 = 2\{E \mathcal{A}_1\} + \{E \mathcal{E}\}$
D_{4d}, S_8	$E_1^3 = \{E_1 + E_3 \mathcal{A}_1\} + \{E_1 \mathcal{E}\}$ $E_2^3 = 2\{E_2 \mathcal{A}_1\} + \{E_2 \mathcal{E}\}$ $E_3^3 = \{E_1 + E_3 \mathcal{A}_1\} + \{E_3 \mathcal{E}\}$
C_5, C_{5v}, D_5	$E_1^3 = \{E_1 + E_2 \mathcal{A}_1\} + \{E_1 \mathcal{E}\}$ $E_2^3 = \{E_1 + E_2 \mathcal{A}_1\} + \{E_2 \mathcal{E}\}$
C_{5h}, D_{5h}	$E_1'^3 = \{E_1' + E_2' \mathcal{A}_1\} + \{E_1' \mathcal{E}\}$ $E_2'^3 = \{E_1' + E_2' \mathcal{A}_1\} + \{E_2' \mathcal{E}\}$ $E_1''^3 = \{E_1'' + E_2'' \mathcal{A}_1\} + \{E_1'' \mathcal{E}\}$ $E_2''^3 = \{E_1'' + E_2'' \mathcal{A}_1\} + \{E_2'' \mathcal{E}\}$
C_6	$E_1^3 = \{2B + E_1 \mathcal{A}_1\} + \{E_1 \mathcal{E}\}$ $E_2^3 = \{2A + E_2 \mathcal{A}_1\} + \{E_2 \mathcal{E}\}$
C_{6v}, D_6	$E_1^3 = \{B_1 + B_2 + E_1 \mathcal{A}_1\} + \{E_1 \mathcal{E}\}$ $E_2^3 = \{A_1 + A_2 + E_2 \mathcal{A}_1\} + \{E_2 \mathcal{E}\}$
D_{6d}	$E_1^3 = \{E_1 + E_3 \mathcal{A}_1\} + \{E_1 \mathcal{E}\}$ $E_2^3 = \{B_1 + B_2 + E_2 \mathcal{A}_1\} + \{E_2 \mathcal{E}\}$ $E_3^3 = 2\{E_3 \mathcal{A}_1\} + \{E_3 \mathcal{E}\}$ $E_4^3 = \{A_1 + A_2 + E_4 \mathcal{A}_1\} + \{E_4 \mathcal{E}\}$ $E_5^3 = \{E_3 + E_5 \mathcal{A}_1\} + \{E_5 \mathcal{E}\}$
S_{12}	$E_1^3 = \{E_1 + E_3 \mathcal{A}_1\} + \{E_1 \mathcal{E}\}$ $E_2^3 = \{2B + E_2 \mathcal{A}_1\} + \{E_2 \mathcal{E}\}$ $E_3^3 = 2\{E_3 \mathcal{A}_1\} + \{E_3 \mathcal{E}\}$ $E_4^3 = \{2A + E_4 \mathcal{A}_1\} + \{E_4 \mathcal{E}\}$ $E_5^3 = \{E_3 + E_5 \mathcal{A}_1\} + \{E_5 \mathcal{E}\}$
$C_{\infty v}, D_{\infty}$	$E_n^3 = \{E_n + E_{3n} \mathcal{A}_1\} + \{E_n \mathcal{E}\} \quad (E_1 \equiv \Pi, E_2 \equiv \Delta, \dots)$
T	$E^3 = \{2A + E \mathcal{A}_1\} + \{E \mathcal{E}\}$ $T^3 = \{A + 3T \mathcal{A}_1\} + \{A \mathcal{A}_2\} + \{E + 2T \mathcal{E}\}$
$T_d O,$	$E^3 = \{A_1 + A_2 + E \mathcal{A}_1\} + \{E \mathcal{E}\}$ $T_1^3 = \{A_2 + 2T_1 + T_2 \mathcal{A}_1\} + \{A_1 \mathcal{A}_2\} + \{E + T_1 + T_2 \mathcal{E}\}$ $T_2^3 = \{A_1 + T_1 + 2T_2 \mathcal{A}_1\} + \{A_1 \mathcal{A}_2\} + \{E + T_1 + T_2 \mathcal{E}\}$

(Continued)

TABLE II. (Continued)

<i>I</i>	$T_1^3 = \{T_1 + T_2 + G \mid \mathcal{A}_1\} + \{A \mid \mathcal{A}_2\} + \{T_1 + H \mid \mathcal{E}\}$
	$T_2^3 = \{T_1 + T_2 + G \mid \mathcal{A}_1\} + \{A \mid \mathcal{A}_2\} + \{T_2 + H \mid \mathcal{E}\}$
	$G^3 = \{A + T_1 + T_2 + 2G + H \mid \mathcal{A}_1\} + \{G \mid \mathcal{A}_2\}$ $+ \{T_1 + T_2 + G + 2H \mid \mathcal{E}\}$
	$H^3 = \{2A + T_1 + T_2 + 3G + 3H \mid \mathcal{A}_1\} + \{T_1 + T_2 + G \mid \mathcal{A}_2\}$ $+ \{2T_1 + 2T_2 + 2G + 4H \mid \mathcal{E}\}$
<i>K</i>	$D_1^3 = \{D_1 + D_3 \mid \mathcal{A}_1\} + \{D_0 \mid \mathcal{A}_2\} + \{D_1 + D_2 \mid \mathcal{E}\}$
	$D_2^3 = \{D_0 + D_2 + D_3 + D_4 + D_6 \mid \mathcal{A}_1\} + \{D_1 + D_3 \mid \mathcal{A}_2\}$ $+ \{D_1 + 2D_2 + D_3 + D_4 + D_5 \mid \mathcal{E}\}$

TABLE III. The symmetrized fourth powers of the irreducible representations of non-centrosymmetric point groups.

C_3	$E^4 = \{A + 2E \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E \mathcal{T}_2\}$
C_{3v}, D_3	$E^4 = \{A_1 + 2E \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E \mathcal{T}_2\}$
C_{3h}	$E'^4 = E''^4 = \{A' + 2E' \mathcal{A}_1\} + \{A' \mathcal{E}\} + \{A' + E' \mathcal{T}_2\}$
D_{3h}	$E'^4 = E''^4 = \{A'_1 + 2E' \mathcal{A}_1\} + \{A'_1 \mathcal{E}\} + \{A'_2 + E' \mathcal{T}_2\}$
C_4, S_4	$E^4 = \{3A + 2B \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + 2B \mathcal{T}_2\}$
C_{4v}, D_4, D_{2d}	$E^4 = \{2A_1 + A_2 + B_1 + B_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + B_1 + B_2 \mathcal{T}_2\}$
D_{4d}	$E_1^4 = E_3^4 = \{A_1 + B_1 + B_2 + E_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_2 \mathcal{T}_2\}$
	$E_2^4 = \{2A_1 + A_2 + B_1 + B_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + B_1 + B_2 \mathcal{T}_2\}$
S_8	$E_1^4 = E_3^4 = \{A + 2B + E_2 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_2 \mathcal{T}_2\}$
	$E_2^4 = \{3A + 2B \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + 2B \mathcal{T}_2\}$
C_5	$E_1^4 = \{A + E_1 + E_2 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_2 \mathcal{T}_2\}$
	$E_2^4 = \{A + E_1 + E_2 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_1 \mathcal{T}_2\}$
C_{5v}, D_5	$E_1^4 = \{A_1 + E_1 + E_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_2 \mathcal{T}_2\}$
	$E_2^4 = \{A_1 + E_1 + E_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_1 \mathcal{T}_2\}$
C_{5h}	$E_1'^4 = E_1''^4 = \{A' + E_1' + E_2' \mathcal{A}_1\} + \{A' \mathcal{E}\} + \{A' + E_2' \mathcal{T}_2\}$
	$E_2'^4 = E_2''^4 = \{A' + E_1' + E_2' \mathcal{A}_1\} + \{A' \mathcal{E}\} + \{A' + E_1' \mathcal{T}_2\}$
D_{5h}	$E_1'^4 = E_1''^4 = \{A'_1 + E_1' + E_2' \mathcal{A}_1\} + \{A'_1 \mathcal{E}\} + \{A'_2 + E_2' \mathcal{T}_2\}$
	$E_2'^4 = E_2''^4 = \{A'_1 + E_1' + E_2' \mathcal{A}_1\} + \{A'_1 \mathcal{E}\} + \{A'_2 + E_1' \mathcal{T}_2\}$
C_6	$E_1^4 = E_2^4 = \{A + 2E_2 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_2 \mathcal{T}_2\}$
C_{6v}, D_6	$E_1^4 = E_2^4 = \{A_1 + 2E_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_2 \mathcal{T}_2\}$
D_{6d}	$E_1^4 = E_5^4 = \{A_1 + E_2 + E_4 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_2 \mathcal{T}_2\}$
	$E_2^4 = E_4^4 = \{A_1 + 2E_4 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_4 \mathcal{T}_2\}$
	$E_3^4 = \{2A_1 + A_2 + B_1 + B_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + B_1 + B_2 \mathcal{T}_2\}$
S_{12}	$E_1^4 = E_5^4 = \{A + E_2 + E_4 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_2 \mathcal{T}_2\}$
	$E_2^4 = E_4^4 = \{A + 2E_4 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_4 \mathcal{T}_2\}$
	$E_3^4 = \{3A + 2B \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + 2B \mathcal{T}_2\}$
$C_{\infty v}, D_{\infty}$	$E_n^4 = \{A_1 + E_{2n} + E_{4n} \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_{2n} \mathcal{T}_2\}$
T	$E^4 = \{A + 2E \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E \mathcal{T}_2\}$
	$T_4 = \{2A + 2E + 3T \mathcal{A}_1\} + \{A + E + T \mathcal{E}\} + \{T \mathcal{T}_1\}$
	$\quad + \{A + E + 4T \mathcal{T}_2\}$
T_d, O	$E^4 = \{A_1 + 2E \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E \mathcal{T}_2\}$
	$T_1^4 = T_2^4 = \{2A_1 + 2E + T_1 + 2T_2 \mathcal{A}_1\} + \{A_1 + E + T_2 \mathcal{E}\}$
	$\quad + \{T_1 \mathcal{T}_1\} + \{A_2 + E + 2T_1 + 2T_2 \mathcal{T}_2\}$
I	$T_1^4 = \{A + G + 2H \mathcal{A}_1\} + \{A + H \mathcal{E}\} + \{T_1 \mathcal{T}_1\}$
	$\quad + \{T_1 + T_2 + G + H \mathcal{T}_2\}$
	$T_2^4 = \{A + G + 2H \mathcal{A}_1\} + \{A + H \mathcal{E}\} + \{T_2 \mathcal{T}_1\}$
	$\quad + \{T_1 + T_2 + G + H \mathcal{T}_2\}$

(Continued)

TABLE III. (Continued)

	$G^4 = \{2A + T_1 + T_2 + 3G + 4H \mid \mathcal{A}_1\} + \{A \mid \mathcal{A}_2\}$ $+ \{A + G + 3H \mid \mathcal{E}\} + \{T_1 + T_2 + G + H \mid \mathcal{F}_1\}$ $+ 3\{T_1 + T_2 + G + H \mid \mathcal{F}_2\}$
	$H^4 = \{2A + 2T_1 + 2T_2 + 4G + 8H \mid \mathcal{A}_1\} + \{H \mid \mathcal{A}_2\}$ $+ \{3A + T_1 + T_2 + 4G + 5H \mid \mathcal{E}\} + \{3T_1 + 3T_2 + 3G$ $+ 3H \mid \mathcal{F}_1\} + \{A + 6T_1 + 6T_2 + 7G + 8H \mid \mathcal{F}_2\}$
K	$D_1^4 = \{D_0 + D_2 + D_4 \mid \mathcal{A}_1\} + \{D_0 + D_2 \mid \mathcal{E}\} + \{D_1 \mid \mathcal{F}_1\}$ $+ \{D_1 + D_2 + D_3 \mid \mathcal{F}_2\}$
	$D_2^4 = \{D_0 + 2D_2 + 2D_4 + D_5 + D_6 + D_8 \mid \mathcal{A}_1\} + \{D_2 \mid \mathcal{A}_2\}$ $+ \{2D_0 + 2D_2 + D_3 + 2D_4 + D_6 \mid \mathcal{E}\}$ $+ \{2D_1 + D_2 + 2D_3 + D_4 + D_5 \mid \mathcal{F}_1\} + \{2D_1 + 3D_3 + 2D_4$ $+ 2D_5 + D_6 + D_7 \mid \mathcal{F}_2\}$

TABLE IV. The symmetrized fifth powers of the irreducible representations of non-centrosymmetric point groups.

C_3	$E^5 = 2\{A + E \mathcal{A}_1\} + \{2A + E \mathcal{G}_1\} + \{E \mathcal{H}_1\}$
C_{3v}, D_3	$E^5 = \{A_1 + A_2 + 2E \mathcal{A}_1\} + \{A_1 + A_2 + E \mathcal{G}_1\} + \{E \mathcal{H}_1\}$
C_{3h}	$E_1^5 = 2\{A' + E' \mathcal{A}_1\} + \{2A' + E' \mathcal{G}_1\} + \{E' \mathcal{A}_1\}$ $E''^5 = 2\{A'' + E'' \mathcal{A}_1\} + \{2A'' + E'' \mathcal{G}_1\} + \{E'' \mathcal{H}_1\}$
D_{3h}	$E_1^5 = \{A'_1 + A'_2 + 2E' \mathcal{A}_1\} + \{A'_1 + A'_2 + E' \mathcal{G}_1\} + \{E' \mathcal{H}_1\}$ $E''^5 = \{A''_1 + A''_2 + 2E'' \mathcal{A}_1\} + \{A''_1 + A''_2 + E'' \mathcal{G}_1\} + \{E'' \mathcal{H}_1\}$
$C_4, C_{4v}, D_4, D_{2d}, S_4, D_{4d}, S_8$	$E^5 = 3\{E \mathcal{A}_1\} + 2\{E \mathcal{G}_1\} + \{E \mathcal{H}_1\}$ $E_1^5 = \{E_1 + 2E_3 \mathcal{A}_1\} + \{E_1 + E_3 \mathcal{G}_1\} + \{E_1 \mathcal{H}_1\}$ $E_2^5 = 3\{E_2 \mathcal{A}_1\} + 2\{E_2 \mathcal{G}_1\} + \{E_2 \mathcal{H}_1\}$ $E_3^5 = \{2E_1 + E_3 \mathcal{A}_1\} + \{E_1 + E_3 \mathcal{G}_1\} + \{E_3 \mathcal{H}_1\}$
C_5	$E_1^5 = \{2A + E_1 + E_2 \mathcal{A}_1\} + \{E_1 + E_2 \mathcal{G}_1\} + \{E_1 \mathcal{H}_1\}$ $E_2^5 = \{2A + E_1 + E_2 \mathcal{A}_1\} + \{E_1 + E_2 \mathcal{G}_1\} + \{E_2 \mathcal{H}_1\}$
C_{5v}, D_5	$E_1^5 = \{A_1 + A_2 + E_1 + E_2 \mathcal{A}_1\} + \{E_1 + E_2 \mathcal{G}_1\} + \{E_1 \mathcal{H}_1\}$ $E_2^5 = \{A_1 + A_2 + E_1 + E_2 \mathcal{A}_1\} + \{E_1 + E_2 \mathcal{G}_1\} + \{E_2 \mathcal{H}_1\}$
C_{5h}	$E_1^5 = \{2A' + E'_1 + E'_2 \mathcal{A}_1\} + \{E'_1 + E'_2 \mathcal{G}_1\} + \{E'_1 \mathcal{H}_1\}$ $E_2^5 = \{2A' + E'_1 + E'_2 \mathcal{A}_1\} + \{E'_1 + E'_2 \mathcal{G}_1\} + \{E'_2 \mathcal{H}_1\}$ $E_1''^5 = \{2A'' + E''_1 + E''_2 \mathcal{A}_1\} + \{E''_1 + E''_2 \mathcal{G}_1\} + \{E''_1 \mathcal{H}_1\}$ $E_2''^5 = \{2A'' + E''_1 + E''_2 \mathcal{A}_1\} + \{E''_1 + E''_2 \mathcal{G}_1\} + \{E''_2 \mathcal{H}_1\}$
D_{5h}	$E_1^5 = \{A'_1 + A'_2 + E'_1 + E'_2 \mathcal{A}_1\} + \{E'_1 + E'_2 \mathcal{G}_1\} + \{E'_1 \mathcal{H}_1\}$ $E_2^5 = \{A'_1 + A'_2 + E'_1 + E'_2 \mathcal{A}_1\} + \{E'_1 + E'_2 \mathcal{G}_1\} + \{E'_2 \mathcal{H}_1\}$ $E_1''^5 = \{A''_1 + A''_2 + E''_1 + E''_2 \mathcal{A}_1\} + \{E''_1 + E''_2 \mathcal{G}_1\} + \{E''_1 \mathcal{H}_1\}$ $E_2''^5 = \{A''_1 + A''_2 + E''_1 + E''_2 \mathcal{A}_1\} + \{E''_1 + E''_2 \mathcal{G}_1\} + \{E''_2 \mathcal{H}_1\}$
C_6	$E_1^5 = 2\{B + E_1 \mathcal{A}_1\} + \{2B + E_1 \mathcal{G}_1\} + \{E_1 \mathcal{H}_1\}$ $E_2^5 = 2\{A + E_2 \mathcal{A}_1\} + \{2A + E_2 \mathcal{G}_1\} + \{E_2 \mathcal{H}_1\}$
C_{6v}, D_6	$E_1^5 = \{B_1 + B_2 + 2E_1 \mathcal{A}_1\} + \{B_1 + B_2 + E_1 \mathcal{G}_1\} + \{E_1 \mathcal{H}_1\}$ $E_2^5 = \{A_1 + A_2 + 2E_2 \mathcal{A}_1\} + \{A_1 + A_2 + E_2 \mathcal{G}_1\} + \{E_2 \mathcal{H}_1\}$
D_{6d}	$E_1^5 = \{E_1 + E_3 + E_5 \mathcal{A}_1\} + \{E_1 + E_3 \mathcal{G}_1\} + \{E_1 \mathcal{H}_1\}$ $E_2^5 = \{B_1 + B_2 + 2E_2 \mathcal{A}_1\} + \{B_1 + B_2 + E_2 \mathcal{G}_1\} + \{E_2 \mathcal{H}_1\}$ $E_3^5 = 3\{E_3 \mathcal{A}_1\} + 2\{E_3 \mathcal{G}_1\} + \{E_3 \mathcal{H}_1\}$ $E_4^5 = \{A_1 + A_2 + 2E_4 \mathcal{A}_1\} + \{A_1 + A_2 + E_4 \mathcal{G}_1\} + \{E_4 \mathcal{H}_1\}$ $E_5^5 = \{E_1 + E_3 + E_5 \mathcal{A}_1\} + \{E_3 + E_5 \mathcal{G}_1\} + \{E_5 \mathcal{H}_1\}$
S_{12}	$E_1^5 = \{E_1 + E_3 + E_5 \mathcal{A}_1\} + \{E_1 + E_3 \mathcal{G}_1\} + \{E_1 \mathcal{H}_1\}$ $E_2^5 = 2\{B + E_2 \mathcal{A}_1\} + \{2B + E_2 \mathcal{G}_1\} + \{E_2 \mathcal{H}_1\}$ $E_3^5 = 3\{E_3 \mathcal{A}_1\} + 2\{E_3 \mathcal{G}_1\} + \{E_3 \mathcal{H}_1\}$ $E_4^5 = 2\{A + E_4 \mathcal{A}_1\} + \{2A + E_4 \mathcal{G}_1\} + \{E_4 \mathcal{H}_1\}$ $E_5^5 = \{E_1 + E_3 + E_5 \mathcal{A}_1\} + \{E_3 + E_5 \mathcal{G}_1\} + \{E_5 \mathcal{H}_1\}$
$C_{\infty v}, D_{\infty}$	$E_n^5 = \{E_n + E_{3n} + E_{5n} \mathcal{A}_1\} + \{E_n + E_{3n} \mathcal{G}_1\} + \{E_n \mathcal{H}_1\}$

(Continued)

TABLE IV. (Continued)

T	$E^5 = 2\{A + E \mathcal{A}_1\} + \{2A + E \mathcal{G}_1\} + \{E \mathcal{H}_1\}$ $T^5 = \{A + E + 6T \mathcal{A}_1\} + 2\{A + E + 3T \mathcal{G}_1\}$ $+ \{A + E + 4T \mathcal{H}_1\} + \{T \mathcal{H}_2\} + \{A + E + T \mathcal{I}\}$
T_d, O	$E^5 = \{A_1 + A_2 + 2E \mathcal{A}_1\} + \{A_1 + A_2 + E \mathcal{G}_1\} + \{E \mathcal{H}_1\}$ $T_1^5 = \{A_2 + E + 4T_1 + 2T_2 \mathcal{A}_1\}$ $+ \{A_1 + A_2 + 2E + 3T_1 + 3T_2 \mathcal{G}_1\}$ $+ \{A_2 + E + 2T_1 + 2T_2 \mathcal{H}_1\} + \{T_1 \mathcal{H}_1\} + \{A_1 + E + T_2 \mathcal{I}\}$ $T_2^5 = \{A_1 + E + 2T_1 + 4T_2 \mathcal{A}_1\}$ $+ \{A_1 + A_2 + 2E + 3T_1 + 3T_2 \mathcal{G}_1\} + \{T_2 \mathcal{H}_1\}$ $+ \{A_2 + E + T_1 \mathcal{I}\}$
I	$T_1^5 = \{2T_1 + 2T_2 + G + H \mathcal{A}_1\} + \{T_1 + T_2 + 2G + 2H \mathcal{G}_1\}$ $+ \{T_1 + T_2 + G + H \mathcal{H}_1\} + \{T_1 \mathcal{H}_2\} + \{A + H \mathcal{I}\}$ $T_2^5 = \{2T_1 + 2T_2 + G + H \mathcal{A}_1\} + \{T_1 + T_2 + 2G + 2H \mathcal{G}_1\}$ $+ \{T_1 + T_2 + G + H \mathcal{H}_1\} + \{T_2 \mathcal{H}_2\} + \{A + H \mathcal{I}\}$ $G^5 = \{2A + 3T_1 + 3T_2 + 4G + 4H \mathcal{A}_1\}$ $+ \{A + 4T_1 + 4T_2 + 6G + 7H \mathcal{G}_1\} + \{G \mathcal{G}_2\}$ $+ \{A + 3T_1 + 3T_2 + 4G + 5H \mathcal{H}_1\}$ $+ \{T_1 + T_2 + G + 2H \mathcal{H}_2\}$ $+ \{A + 2T_1 + 2T_2 + 2G + 3H \mathcal{I}\}$ $H^5 = \{4A + 5T_1 + 5T_2 + 8G + 12H \mathcal{A}_1\} + \{A \mathcal{A}_2\}$ $+ \{4A + 11T_1 + 11T_2 + 16G + 18H \mathcal{G}_1\} + \{T_1 + T_2 + 2G$ $+ 2H \mathcal{G}_2\} + \{3A + 8T_1 + 8T_2 + 11G + 16H \mathcal{H}_1\}$ $+ \{2A + 3T_1 + 3T_2 + 5G + 7H \mathcal{H}_2\}$ $+ \{A + 8T_1 + 8T_2 + 8G + 9H \mathcal{I}\}$
K	$D_1^5 = \{D_1 + D_3 \mathcal{A}_1\} + \{D_1 + D_2 + D_3 + D_4 \mathcal{G}_1\}$ $+ \{D_1 + D_2 + D_3 \mathcal{H}_1\} + \{D_1 \mathcal{H}_2\} + \{D_0 + D_2 \mathcal{I}\}$ $D_2^5 = \{D_0 + 2D_2 + D_3 + 2D_4 + D_5 + 2D_6 + D_7 + D_8 + D_{10} \mathcal{A}_1\}$ $+ \{D_0 \mathcal{A}_2\} + \{D_0 + 2D_1 + 3D_2 + 4D_3 + 4D_4 + 3D_5 + 3D_6$ $+ 2D_7 + D_8 + D_9 \mathcal{G}_1\} + \{D_1 + D_2 + D_3 + D_4 \mathcal{G}_2\} + \{D_0$ $+ 2D_1 + 4D_2 + 3D_3 + 4D_4 + 3D_5 + 2D_6 + D_7 + D_8 \mathcal{H}_1\}$ $+ \{D_0 + D_1 + 3D_2 + 2D_3 + 2D_4 + D_5 + D_6 \mathcal{H}_2\}$ $+ \{3D_1 + 2D_2 + 4D_3 + 2D_4 + 3D_5 + D_6 + D_7 \mathcal{I}\}$ <p>.....</p>

TABLE V. The symmetrized cubes of the double-valued representations of non-centrosymmetric double groups.

$C'_{1h}, C'_2, C'_{2v}, D'_2$	$E^3_{1/2} = 2\{E_{1/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$
C'_3	$E^3_{1/2} = \{2E_{3/2} + E_{1/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$
C'_{3v}, D'_3	$E^3_{1/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$ $E^3_{3/2} = 2\{E_{3/2} \mathcal{A}_1\} + \{E_{3/2} \mathcal{E}\}$
$C'_4, C'_{4v}, D'_4, S'_4, D'_{2d}$	$E^3_{1/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$ $E^3_{3/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{3/2} \mathcal{E}\}$
D'_{4d}, S'_8	$E^3_{1/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$ $E^3_{3/2} = \{E_{3/2} + E_{7/2} \mathcal{A}_1\} + \{E_{3/2} \mathcal{E}\}$ $E^3_{5/2} = \{E_{1/2} + E_{5/2} \mathcal{A}_1\} + \{E_{5/2} \mathcal{E}\}$ $E^3_{7/2} = \{E_{5/2} + E_{7/2} \mathcal{A}_1\} + \{E_{7/2} \mathcal{E}\}$
C'_5	$E^3_{1/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$ $E^3_{3/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{3/2} \mathcal{E}\}$
C'_{5v}, D'_5	$E^3_{1/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$ $E^3_{3/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{3/2} \mathcal{E}\}$ $E^3_{5/2} = 2\{E_{5/2} \mathcal{A}_1\} + \{E_{5/2} \mathcal{E}\}$
C'_{6h}, D'_{6h}	$E^3_{1/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$ $E^3_{3/2} = \{E_{3/2} + E_{9/2} \mathcal{A}_1\} + \{E_{3/2} \mathcal{E}\}$ $E^3_{5/2} = 2\{E_{5/2} \mathcal{A}_1\} + \{E_{5/2} \mathcal{E}\}$ $E^3_{7/2} = \{E_{1/2} + E_{7/2} \mathcal{A}_1\} + \{E_{7/2} \mathcal{E}\}$ $E^3_{9/2} = \{E_{7/2} + E_{9/2} \mathcal{A}_1\} + \{E_{9/2} \mathcal{E}\}$
$C'_6, C'_{6v}, D'_6, C'_{3h}, D'_{3h}$	$E^3_{1/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$ $E^3_{3/2} = 2\{E_{3/2} \mathcal{A}_1\} + \{E_{3/2} \mathcal{E}\}$ $E^3_{5/2} = \{E_{3/2} + E_{5/2} \mathcal{A}_1\} + \{E_{5/2} \mathcal{E}\}$
D'_{6d}, S'_{12}	$E^3_{1/2} = \{E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$ $E^3_{3/2} = \{E_{3/2} + E_{9/2} \mathcal{A}_1\} + \{E_{3/2} \mathcal{E}\}$ $E^3_{5/2} = \{E_{5/2} + E_{9/2} \mathcal{A}_1\} + \{E_{5/2} \mathcal{E}\}$ $E^3_{7/2} = \{E_{3/2} + E_{7/2} \mathcal{A}_1\} + \{E_{7/2} \mathcal{E}\}$ $E^3_{9/2} = \{E_{3/2} + E_{9/2} \mathcal{A}_1\} + \{E_{9/2} \mathcal{E}\}$ $E^3_{11/2} = \{E_{9/2} + E_{11/2} \mathcal{A}_1\} + \{E_{11/2} \mathcal{E}\}$
$C'_{\infty v}, D'_{\infty}$	$E^3_{r/2} = \{E_{r/2} + E_{3r/2} \mathcal{A}_1\} + \{E_{r/2} \mathcal{E}\} \quad (r = 1, 3, 5, 7, \dots)$
T'	$E^3_{1/2} = \{G_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$ $G^3_{3/2} = 2\{E_{1/2} + 2G_{3/2} \mathcal{A}_1\} + \{G_{3/2} \mathcal{A}_2\} + \{4E_{1/2} + 3G_{3/2} \mathcal{E}\}$
T'_d, O'	$E^3_{1/2} = \{G_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$ $E^3_{5/2} = \{G_{3/2} \mathcal{A}_1\} + \{E_{5/2} \mathcal{E}\}$ $G^3_{3/2} = \{E_{1/2} + E_{5/2} + 4G_{3/2} \mathcal{A}_1\} + \{G_{3/2} \mathcal{A}_2\} + \{2E_{1/2} + 2E_{5/2} + 3G_{3/2} \mathcal{E}\}$

(Continued)

TABLE V. (Continued)

I'	$E_{1/2}^3 = \{G_{3/2} \mathcal{A}_1\} + \{E_{1/2} \mathcal{E}\}$
	$E_{7/2}^3 = \{G_{3/2} \mathcal{A}_1\} + \{E_{7/2} \mathcal{E}\}$
	$G_{3/2}^3 = 2\{G_{3/2} + I_{5/2} \mathcal{A}_1\} + \{G_{3/2} \mathcal{A}_2\} + \{E_{1/2} + E_{7/2} + G_{3/2} + 2I_{5/2} \mathcal{E}\}$
	$I_{5/2}^3 = \{E_{1/2} + E_{7/2} + 4G_{3/2} + 6I_{5/2} \mathcal{A}_1\} + 2\{G_{3/2} + I_{5/2} \mathcal{A}_2\} + \{3E_{1/2} + 3E_{7/2} + 4G_{3/2} + 7I_{5/2} \mathcal{E}\}$

K'	$D_{1/2}^3 = \{D_{3/2} \mathcal{A}_1\} + \{D_{1/2} \mathcal{E}\}$
	$D_{3/2}^3 = \{D_{3/2} + D_{5/2} + D_{9/2} \mathcal{A}_1\} + \{D_{3/2} \mathcal{A}_2\} + \{D_{1/2} + D_{3/2} + D_{5/2} + D_{7/2} \mathcal{E}\}$
	$D_{5/2}^3 = \{D_{3/2} + D_{5/2} + D_{7/2} + D_{9/2} + D_{11/2} + D_{15/2} \mathcal{A}_1\} + \{D_{3/2} + D_{5/2} + D_{9/2} \mathcal{A}_2\} + \{D_{1/2} + D_{3/2} + 2D_{5/2} + 2D_{7/2} + D_{9/2} + D_{11/2} + D_{13/2} \mathcal{E}\}$

TABLE VI. The symmetrized fourth powers of the double-valued representations of non-centrosymmetric point groups.

C'_{1h}	$E^4_{1/2} = \{3A' + 2A'' \mathcal{A}_1\} + \{A' \mathcal{E}\} + \{A' + 2A'' \mathcal{T}_2\}$
C'_2	$E^4_{1/2} = \{3A + 2B \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + 2B \mathcal{T}_2\}$
C'_{2v}	$E^4_{1/2} = \{2A_1 + A_2 + B_1 + B_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + B_1 + B_2 \mathcal{T}_2\}$
D'_2	$E^4_{1/2} = \{2A + B_1 + B_2 + B_3 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{B_1 + B_2 + B_3 \mathcal{T}_2\}$
C'_3	$E^4_{1/2} = \{A + 2E \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E \mathcal{T}_2\}$
C'_{3v}, D'_3	$E^4_{1/2} = \{A_1 + 2E \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E \mathcal{T}_2\}$ $E^4_{3/2} = \{3A_1 + 2A_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_1 + 2A_2 \mathcal{T}_2\}$
C'_{3h}	$E^4_{1/2} = E^4_{5/2} = \{A' + E' + E'' \mathcal{A}_1\} + \{A'_1 \mathcal{E}\} + \{A'_2 + E'' \mathcal{T}_2\}$ $E^4_{3/2} = \{2A'_1 + A''_1 + A'_2 + A''_2 \mathcal{A}_1\} + \{A'_1 \mathcal{E}\} + \{A''_1 + A'_2 + A''_2 \mathcal{T}_2\}$
C'_4, S'_4	$E^4_{1/2} = E^4_{3/2} = \{A + 2B + E \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E \mathcal{T}_2\}$
C'_{4v}, D'_4, D'_{2d}	$E^4_{1/2} = E^4_{3/2} = \{A_1 + B_1 + B_2 + E \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E \mathcal{T}_2\}$
D'_{4d}	$E^4_{1/2} = E^4_{7/2} = \{A_1 + E_2 + E_3 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_3 \mathcal{T}_2\}$ $E^4_{3/2} = E^4_{5/2} = \{A_1 + E_1 + E_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_1 \mathcal{T}_2\}$
S'_8	$E^4_{1/2} = E^4_{7/2} = \{A + E_1 + E_2 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_1 \mathcal{T}_2\}$ $E^4_{3/2} = E^4_{5/2} = \{A + E_2 + E_3 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_3 \mathcal{T}_2\}$
C'_5	$E^4_{1/2} = \{A + E_1 + E_2 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_1 \mathcal{T}_2\}$ $E^4_{3/2} = \{A + E_1 + E_2 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_2 \mathcal{T}_2\}$ $E^4_{5/2} = 5\{A \mathcal{A}_1\} + \{A \mathcal{E}\} + 3\{A \mathcal{T}_2\}$
C'_{5v}, D'_5	$E^4_{1/2} = \{A_1 + E_1 + E_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_1 \mathcal{T}_2\}$ $E^4_{3/2} = \{A_1 + E_1 + E_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_2 \mathcal{T}_2\}$ $E^4_{5/2} = \{3A_1 + 2A_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_1 + 2A_2 \mathcal{T}_2\}$
C'_{5h}	$E^4_{1/2} = E^4_{9/2} = \{A' + E''_1 + E'_2 \mathcal{A}_1\} + \{A' \mathcal{E}\} + \{A' + E''_1 \mathcal{T}_2\}$ $E^4_{3/2} = E^4_{7/2} = \{A' + E'_1 + E''_2 \mathcal{A}_1\} + \{A' \mathcal{E}\} + \{A' + E''_2 \mathcal{T}_2\}$ $E^4_{5/2} = \{3A' + 2A'' \mathcal{A}_1\} + \{A' \mathcal{E}\} + \{A' + 2A'' \mathcal{T}_2\}$
D'_{5h}	$E^4_{1/2} = E^4_{9/2} = \{A'_1 + E''_1 + E'_2 \mathcal{A}_1\} + \{A'_1 \mathcal{E}\} + \{A'_1 + E''_1 \mathcal{T}_2\}$ $E^4_{3/2} = E^4_{7/2} = \{A'_1 + E'_1 + E''_2 \mathcal{A}_1\} + \{A'_1 \mathcal{E}\} + \{A'_1 + E''_2 \mathcal{T}_2\}$ $E^4_{5/2} = \{2A'_1 + A''_1 + A'_2 + A''_2 \mathcal{A}_1\} + \{A'_1 \mathcal{E}\} + \{A'_1 + A''_1 + A''_2 \mathcal{T}_2\}$
C'_6	$E^4_{1/2} = E^4_{5/2} = \{A + E_1 + E_2 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_1 \mathcal{T}_2\}$ $E^4_{3/2} = 2\{A + B \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + 2B \mathcal{T}_2\}$
C'_{6v}, D'_6	$E^4_{1/2} = E^4_{5/2} = \{A_1 + E_1 + E_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_1 \mathcal{T}_2\}$ $E^4_{3/2} = \{2A_1 + A_2 + B_1 + B_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + B_1 + B_2 \mathcal{T}_2\}$

(Continued)

TABLE VI. (Continued)

D'_{6d}	$E_{1/2}^4 = E_{11/2}^4 = \{A_1 + E_2 + E_5 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_5 \mathcal{T}_2\}$ $E_{3/2}^4 = E_{9/2}^4 = \{A_1 + B_1 + B_2 + E_3 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_3 \mathcal{T}_2\}$
S'_{12}	$E_{5/2}^4 = E_{7/2}^4 = \{A_1 + E_1 + E_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_1 \mathcal{T}_2\}$ $E_{1/2}^4 = E_{11/2}^4 = \{A + E_1 + E_2 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_1 \mathcal{T}_2\}$ $E_{3/2}^4 = E_{9/2}^4 = \{A + 2B + E_3 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_3 \mathcal{T}_2\}$ $E_{5/2}^4 = E_{7/2}^4 = \{A + E_2 + E_5 \mathcal{A}_1\} + \{A \mathcal{E}\} + \{A + E_5 \mathcal{T}_2\}$
$C'_{\infty v}, D'_{\infty}$	$E_{2(n+1)/2}^4 = \{A_1 + E_{2n+1} + E_{4n+2} \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{A_2 + E_{2n+1} \mathcal{T}_2\} \quad (n = 0, 1, 2, \dots)$
T'	$E_{1/2}^4 = \{E + T \mathcal{A}_1\} + \{A \mathcal{E}\} + \{T \mathcal{T}_2\}$ $G_{3/2}^4 = \{5A + 3E + 8T \mathcal{A}_1\} + \{A \mathcal{A}_2\} + \{2A + 3E + 4T \mathcal{E}\} + \{A + E + 4T \mathcal{T}_1\} + 3\{A + E + 4T \mathcal{T}_2\}$
T'_d, O'	$E_{1/2}^4 = E_{5/2}^4 = \{E + T_2 \mathcal{A}_1\} + \{A_1 \mathcal{E}\} + \{T_1 \mathcal{T}_2\}$ $G_{3/2}^4 = \{3A_1 + 2A_2 + 3E + 3T_1 + 5T_2 \mathcal{A}_1\} + \{A_1 \mathcal{A}_2\} + \{2A_1 + 3E + T_1 + 3T_2 \mathcal{E}\} + \{A_2 + E + 2T_1 + 2T_2 \mathcal{T}_1\} + \{A_1 + 2A_2 + 3E + 7T_1 + 5T_2 \mathcal{T}_2\}$
I'	$E_{1/2}^4 = \{H \mathcal{A}_1\} + \{A \mathcal{E}\} + \{T_1 \mathcal{T}_2\}$ $E_{7/2}^4 = \{H \mathcal{A}_1\} + \{A \mathcal{E}\} + \{T_2 \mathcal{T}_2\}$ $G_{3/2}^4 = \{2A + T_1 + T_2 + 3G + 3H \mathcal{A}_1\} + \{A \mathcal{A}_2\} + \{A + G + 3H \mathcal{E}\} + \{T_1 + T_2 + G + H \mathcal{T}_1\} + 3\{T_1 + T_2 + G + H \mathcal{T}_2\}$ $I_{5/2}^4 = \{4A + 5T_1 + 5T_2 + 8G + 12H \mathcal{A}_1\} + \{A + G + 2H \mathcal{A}_2\} + \{4A + 3T + 3T_2 + 7G + 11H \mathcal{E}\} + \{A + 6T_1 + 6T_2 + 7G + 8H \mathcal{T}_1\} + 2\{A + 6T_1 + 6T_2 + 7G + 8H \mathcal{T}_2\}$
K'	$D_{1/2}^4 = \{D_2 \mathcal{A}_1\} + \{D_0 \mathcal{E}\} + \{D_1 \mathcal{T}_2\}$ $D_{3/2}^4 = \{D_0 + D_2 + D_3 + D_4 + D_6 \mathcal{A}_1\} + \{D_0 \mathcal{A}_2\} + \{D_0 + 2D_2 + D_4 \mathcal{E}\} + \{D_1 + D_2 + D_3 \mathcal{T}_1\} + \{2D_1 + D_2 + 2D_3 + D_4 + D_5 \mathcal{T}_2\}$
.....	

TABLE VII. The symmetrized fifth powers of the double-valued representations of non-centrosymmetric double groups.

$C'_{1h}, C'_2, C'_{2v}, D'_2$	$E^5_{1/2} = 3\{E_{1/2} \mathcal{A}_1\} + 2\{E_{1/2} \mathcal{G}_1\} + \{E_{1/2} \mathcal{H}_1\}$
C'_3	$E^5_{1/2} = 2\{B_{3/2} + E_{1/2} \mathcal{A}_1\} + \{2B_{3/2} + E_{1/2}\} \mathcal{G}_1 + \{E_{1/2} \mathcal{H}_1\}$
C'_{3v}, D'_3	$E^5_{3/2} = \{2E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\} + \{E_{1/2} \mathcal{H}_1\}$ $E^5_{3/2} = 3\{E_{3/2} \mathcal{A}_1\} + 2\{E_{3/2} \mathcal{G}_1\} + \{E_{3/2} \mathcal{H}_1\}$
$C'_4, C'_{4v}, C'_4, S'_4, D'_{2d}$	$E^5_{1/2} = \{E_{1/2} + 2E_{3/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\} + \{E_{1/2} \mathcal{H}_1\}$ $E^5_{3/2} = \{2E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\} + \{E_{3/2} \mathcal{H}_1\}$
D'_{4d}, S'_8	$E^5_{1/2} = \{E_{1/2} + E_{3/2} + E_{5/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\}$ $+ \{E_{1/2} \mathcal{H}_1\}$ $E^5_{3/2} = \{E_{1/2} + E_{3/2} + E_{7/2} \mathcal{A}_1\} + \{E_{3/2} + E_{9/2} \mathcal{G}_1\}$ $+ \{E_{3/2} \mathcal{H}_1\}$ $E^5_{5/2} = \{E_{1/2} + E_{5/2} + E_{7/2} \mathcal{A}_1\} + \{E_{1/2} + E_{5/2} \mathcal{G}_1\}$ $+ \{E_{5/2} \mathcal{H}_1\}$ $E^5_{7/2} = \{E_{3/2} + E_{5/2} + E_{7/2} \mathcal{A}_1\} + \{E_{5/2} + E_{7/2} \mathcal{G}_1\}$ $+ \{E_{7/2} \mathcal{H}_1\}$
C'_5	$E^5_{1/2} = \{2B_{5/2} + E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\}$ $+ \{E_{1/2} \mathcal{H}_1\}$ $E^5_{3/2} = \{2B_{5/2} + E_{1/2} + E_{3/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\}$ $+ \{E_{3/2} \mathcal{H}_1\}$
C'_{5v}, D'_5	$E^5_{1/2} = \{E_{1/2} + E_{3/2} + E_{5/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\}$ $+ \{E_{1/2} \mathcal{H}_1\}$ $E^5_{3/2} = \{E_{1/2} + E_{3/2} + E_{5/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\}$ $+ \{E_{3/2} \mathcal{H}_1\}$
C'_{5h}, D'_{5h}	$E^5_{5/2} = 3\{E_{5/2} \mathcal{A}_1\} + 2\{E_{5/2} \mathcal{G}_1\} + \{E_{5/2} \mathcal{H}_1\}$ $E^5_{1/2} = \{E_{1/2} + E_{3/2} + E_{5/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\}$ $+ \{E_{1/2} \mathcal{H}_1\}$ $E^5_{3/2} = \{E_{3/2} + E_{5/2} + E_{9/2} \mathcal{A}_1\} + \{E_{3/2} + E_{9/2} \mathcal{G}_1\}$ $+ \{E_{3/2} \mathcal{H}_1\}$ $E^5_{5/2} = 3\{E_{5/2} \mathcal{A}_1\} + 2\{E_{5/2} \mathcal{G}_1\} + \{E_{5/2} \mathcal{H}_1\}$ $E^5_{7/2} = \{E_{1/2} + E_{5/2} + E_{7/2} \mathcal{A}_1\} + \{E_{1/2} + E_{7/2} \mathcal{G}_1\}$ $+ \{E_{7/2} \mathcal{H}_1\}$ $E^5_{9/2} = \{E_{5/2} + E_{7/2} + E_{9/2} \mathcal{A}_1\} + \{E_{7/2} + E_{9/2} \mathcal{G}_1\}$ $+ \{E_{9/2} \mathcal{H}_1\}$
$C'_6, C'_{6v}, D'_6, C'_{3h}, D'_{3h}$	$E^5_{1/2} = \{E_{1/2} + E_{3/2} + E_{5/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\}$ $+ \{E_{1/2} \mathcal{H}_1\}$ $E^5_{3/2} = 3\{E_{3/2} \mathcal{A}_1\} + 2\{E_{3/2} \mathcal{G}_1\} + \{E_{3/2} \mathcal{H}_1\}$ $E^5_{5/2} = \{E_{1/2} + E_{3/2} + E_{5/2} \mathcal{A}_1\} + \{E_{3/2} + E_{5/2} \mathcal{G}_1\}$ $+ \{E_{5/2} \mathcal{H}_1\}$

(Continued)

TABLE VII. (Continued)

D'_{6d}, S'_{12}	$E_{1/2}^5 = \{E_{1/2} + E_{3/2} + E_{5/2} \mathcal{A}_1\} + \{E_{1/2} + E_{3/2} \mathcal{G}_1\} + \{E_{1/2} \mathcal{H}_1\}$ $E_{3/2}^5 = \{E_{3/2} + 2E_{9/2} \mathcal{A}_1\} + \{E_{3/2} + E_{9/2} \mathcal{G}_1\} + \{E_{3/2} \mathcal{H}_1\}$ $E_{5/2}^5 = \{E_{1/2} + E_{5/2} + E_{9/2} \mathcal{A}_1\} + \{E_{5/2} + E_{9/2} \mathcal{G}_1\} + \{E_{5/2} \mathcal{H}_1\}$ $E_{7/2}^5 = \{E_{3/2} + E_{7/2} + E_{11/2} \mathcal{A}_1\} + \{E_{3/2} + E_{7/2} \mathcal{G}_1\} + \{E_{7/2} \mathcal{H}_1\}$ $E_{9/2}^5 = \{2E_{3/2} + E_{9/2} \mathcal{A}_1\} + \{E_{3/2} + E_{9/2} \mathcal{G}_1\} + \{E_{9/2} \mathcal{H}_1\}$ $E_{11/2}^5 = \{E_{7/2} + E_{9/2} + E_{11/2} \mathcal{A}_1\} + \{E_{9/2} + E_{11/2} \mathcal{G}_1\} + \{E_{11/2} \mathcal{H}_1\}$
$C'_{\infty v}, D'_{\infty}$	$E_{r/2}^5 = \{E_{r/2} + E_{3r/2} + E_{5r/2} \mathcal{A}_1\} + \{E_{r/2} + E_{3r/2} \mathcal{G}_1\} + \{E_{r/2} \mathcal{H}_1\}$ <p style="text-align: right;">($r = 1, 3, 5, 7, \dots$)</p>
T'	$E_{1/2}^5 = \{E_{1/2} + G_{3/2} \mathcal{A}_1\} + \{G_{3/2} \mathcal{G}_1\} + \{E_{1/2} \mathcal{H}_1\}$ $G_{3/2}^5 = 2\{4E_{1/2} + 5G_{3/2} \mathcal{A}_1\} + 14\{E_{1/2} + G_{3/2} \mathcal{G}_1\} + \{G_{3/2} \mathcal{G}_2\} + 10\{E_{1/2} + G_{3/2} \mathcal{H}_1\} + \{4E_{1/2} + 3G_{3/2} \mathcal{H}_2\} + 6\{E_{1/2} + G_{3/2} \mathcal{I}\}$
T'_d, O'	$E_{1/2}^5 = \{E_{5/2} + G_{3/2} \mathcal{A}_1\} + \{G_{3/2} \mathcal{G}_1\} + \{E_{1/2} \mathcal{H}_1\}$ $E_{3/2}^5 = \{E_{1/2} + G_{3/2} \mathcal{A}_1\} + \{G_{3/2} \mathcal{G}_1\} + \{E_{5/2} \mathcal{H}_1\}$ $G_{3/2}^5 = \{4E_{1/2} + 4E_{5/2} + 10G_{3/2} \mathcal{A}_1\} + 7\{E_{1/2} + E_{5/2} + 2G_{3/2} \mathcal{G}_1\} + \{G_{3/2} \mathcal{G}_2\} + 5\{E_{1/2} + E_{5/2} + 2G_{3/2} \mathcal{H}_1\} + \{2E_{1/2} + 2E_{5/2} + 3G_{3/2} \mathcal{H}_2\} + 3\{E_{1/2} + E_{5/2} + 2G_{3/2} \mathcal{I}\}$
I'	$E_{1/2}^5 = \{I_{5/2} \mathcal{A}_1\} + \{G_{3/2} \mathcal{G}_1\} + \{E_{1/2} \mathcal{H}_1\}$ $E_{7/2}^5 = \{I_{5/2} \mathcal{A}_1\} + \{G_{3/2} \mathcal{G}_1\} + \{E_{7/2} \mathcal{H}_1\}$ $G_{3/2}^5 = \{E_{1/2} + E_{7/2} + 4G_{3/2} + 6I_{5/2} \mathcal{A}_1\} + \{3E_{1/2} + 3E_{7/2} + 6G_{3/2} + 8I_{5/2} \mathcal{G}_1\} + \{G_{3/2} \mathcal{G}_2\} + 2\{E_{1/2} + E_{7/2} + 2G_{3/2} + 3I_{5/2} \mathcal{H}_1\} + \{E_{1/2} + E_{7/2} + G_{3/2} + 2I_{5/2} \mathcal{H}_2\} + \{E_{1/2} + E_{7/2} + 2G_{3/2} + 4I_{5/2} \mathcal{I}\}$ $I_{5/2}^5 = 2\{4E_{1/2} + 4E_{7/2} + 8G_{3/2} + 13I_{5/2} \mathcal{A}_1\} + \{I_{5/2} \mathcal{A}_2\} + \{17E_{1/2} + 17E_{7/2} + 34G_{3/2} + 50I_{5/2} \mathcal{G}_1\} + \{3E_{1/2} + 3E_{7/2} + 6G_{3/2} + 8I_{5/2} \mathcal{G}_2\} + 14\{E_{1/2} + E_{7/2} + 2G_{3/2} + 3I_{5/2} \mathcal{H}_1\} + 7\{E_{1/2} + E_{7/2} + 2G_{3/2} + 3I_{5/2} \mathcal{H}_2\} + \{11E_{1/2} + 11E_{7/2} + 22G_{3/2} + 34I_{5/2} \mathcal{I}\}$
K'	$D_{1/2}^5 = \{D_{5/2} \mathcal{A}_1\} + \{D_{3/2} \mathcal{G}_1\} + \{D_{1/2} \mathcal{H}_1\}$

(Continued)

TABLE VII. (Continued)

$$\begin{aligned}
 D_{3/2}^5 = & \{D_{3/2} + D_{5/2} + D_{7/2} + D_{9/2} + D_{11/2} + D_{15/2} \mid \mathcal{A}_1\} \\
 & + \{D_{1/2} + 2D_{3/2} + 2D_{5/2} + 2D_{7/2} + 2D_{9/2} + D_{11/2} \\
 & + D_{13/2} \mid \mathcal{G}_1\} + \{D_{3/2} \mid \mathcal{G}_2\} + \{D_{1/2} + 2D_{3/2} \\
 & + 2D_{5/2} + 2D_{7/2} + D_{9/2} + D_{11/2} \mid \mathcal{H}_1\} \\
 & + \{D_{1/2} + D_{3/2} + D_{5/2} + D_{7/2} \mid \mathcal{H}_2\} \\
 & + \{D_{1/2} + D_{3/2} + 2D_{5/2} + D_{7/2} + D_{9/2} \mid \mathcal{I}\}
 \end{aligned}$$

.....
