

One-dimensional photonic crystal

Maxwell equation 1D:

$$\frac{\partial^2 \varepsilon}{\partial x^2} = \varepsilon(x) \mu(x) \varepsilon_0 \mu_0 \frac{\partial^2 \varepsilon}{\partial t^2}$$

We deal with non-magnetic media, $\mu(r) = 1$, remind that $\varepsilon_0 \mu_0 = 1/c^2$ (where c is the speed of light in a vacuum) and consider stationary waves $\varepsilon(x, t) = E(x)e^{i\omega t}$. The above equation turns into

$$-\frac{1}{\varepsilon(x)} \frac{d^2 E}{dx^2} = \lambda E,$$

with $\lambda = \omega^2 / c^2$.

After discretization

$$-\frac{1}{\varepsilon_i} \frac{1}{h^2} (E_{i+1} + E_{i-1} - 2E_i) = \lambda E_i$$

we apply BCs: $E_0 = e^{-i\theta} E_{n-1}$ and $E_n = e^{+i\theta} E_1$

Maxwell equation 3D:

$$\nabla^2 \varepsilon = \varepsilon(r) \mu(r) \varepsilon_0 \mu_0 \frac{\partial^2 \varepsilon}{\partial t^2}$$

For stationary waves, $\varepsilon(r, t) = E(r)e^{i\omega t}$, and non-magnetic media, the above equation turns into

$$-\frac{1}{\varepsilon(r)} \nabla^2 E = \lambda E,$$

with $\lambda = \omega^2 / c^2$.

Note that $\lambda = \omega^2 / c^2$. Then it is ω and not λ what is represented in the reciprocal lattice. Then, the profile in the surroundings of the Γ point is linear.