Coordenades cartesianes, esfèriques,...

Notació:

x (text pla) representa un escalar

x (en negreta) representa un vector

$\mathbf{u_i}$ (en negreta) representa un vector unitari en la direcció $\partial \mathbf{r}/\partial i$

Tenim que:

$\mathbf{r} = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$

$\partial \mathbf{r}/\partial x = h_x \mathbf{u_x} = \mathbf{i}$

$\partial \mathbf{r}/\partial y = h_y \mathbf{u_y} = \mathbf{j}$

$\partial \mathbf{r}/\partial z = h_z \mathbf{u_z} = \mathbf{k}$

$d\mathbf{r} = (\partial \mathbf{r}/\partial x)\, dx + (\partial \mathbf{r}/\partial y)\, dy + (\partial \mathbf{r}/\partial z)\, dz = dx\,\mathbf{i} + dy\,\mathbf{j} + dz\,\mathbf{k}$

$dv = dx\,\mathbf{i}\,(dy\,\mathbf{j} \wedge dz\,\mathbf{k}) = dx\,dy\,dz\,\mathbf{i}\,(\mathbf{j} \wedge \mathbf{k}) = dx\,dy\,dz$

Perquè: $\mathbf{i}\,(\mathbf{j} \wedge \mathbf{k}) = 1$

Però també :

$\mathbf{r} = r \sin\theta \cos\phi\,\mathbf{i} + r \sin\theta \sin\phi\,\mathbf{j} + r \cos\theta\,\mathbf{k}$

$\partial \mathbf{r}/\partial r = h_r \mathbf{u_r} = \sin\theta \cos\phi\,\mathbf{i} + \sin\theta \sin\phi\,\mathbf{j} + \cos\theta\,\mathbf{k} \quad = \quad \mathbf{u_r}$

$\partial \mathbf{r}/\partial \theta = h_\theta \mathbf{u_\theta} = r\,[\cos\theta \cos\phi\,\mathbf{i} + \cos\theta \sin\phi\,\mathbf{j} - \sin\theta\,\mathbf{k}] = \quad r\,\mathbf{u_\theta}$

$\partial \mathbf{r}/\partial \phi = h_\phi \mathbf{u_\phi} = r \sin\theta\,[-\sin\phi\,\mathbf{i} + \cos\phi\,\mathbf{j}] \quad = \quad r \sin\theta\,\mathbf{u_\phi}$

$d\mathbf{r} = (\partial \mathbf{r}/\partial r)\, dr + (\partial \mathbf{r}/\partial \theta)\, d\theta + (\partial \mathbf{r}/\partial \phi)\, d\phi = dr\,\mathbf{u_r} + r\,d\theta\,\mathbf{u_\theta} + r \sin\theta\, d\phi\,\mathbf{u_\phi}$

$dv = r^2 \sin\theta\, dr\, d\theta\, d\phi\,\mathbf{u_r}\,(\mathbf{u_\theta} \wedge \mathbf{u_\phi}) = r^2 \sin\theta\, dr\, d\theta\, d\phi$

Atès que:

$\mathbf{u_r}\,(\mathbf{u_\theta} \wedge \mathbf{u_\phi}) = (\sin\theta \cos\phi\,\mathbf{i} + \sin\theta \sin\phi\,\mathbf{j} + \cos\theta\,\mathbf{k})\,(\,(\cos\theta \cos\phi\,\mathbf{i} + \cos\theta \sin\phi\,\mathbf{j} - \sin\theta\,\mathbf{k}) \wedge (-\sin\phi\,\mathbf{i} + \cos\phi\,\mathbf{j})) = 1.$