

(* Quoficients per a la derivada central a cinc punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi

$$y = a + b x + c x^2 + d x^3 + e x^4 \quad *)$$

(* Hipòtesi: $y''(i) = aa y(i+2) + bb y(i-1) + cc y(i) + dd y(i-1) + ee y(i-2)$ *)

$$(*) \quad y(i+2) = a + b(2h) + c(2h)^2 + d(2h)^3 + e(2h)^4$$

$$y(i+1) = a + b h + c h^2 + d h^3 + e h^4$$

$$y(i) = a$$

$$y(i-1) = a - b h + c h^2 - d h^3 + e h^4$$

$$y(i-2) = a - b(2h) + c(2h)^2 - d(2h)^3 + e(2h)^4$$

$$y''(i) = 2c$$

EN CONSEQUÈNCIA:

$$\begin{aligned} 2c &= aa(a + 2bh + 4ch^2 + 8dh^3 + 16eh^4) \\ &+ bb(a + bh + ch^2 + dh^3 + eh^4) + cc(a) \\ &+ dd(a - bh + ch^2 - dh^3 + eh^4) \\ &+ ee(a - 2bh + 4ch^2 - 8dh^3 + 16eh^4) \end{aligned} \quad *)$$

$$\text{Solve}[\{(aa + bb + cc + dd + ee) == 0, \\ (2aa + bb - dd - 2ee) == 0, \\ h^2(4aa + bb + dd + 4ee) == 2, \\ (8aa + bb - dd - 8ee) == 0, \\ (16aa + bb + dd + 16ee) == 0\}, \{aa, bb, cc, dd, ee\}]$$

$$\left\{ \left\{ cc \rightarrow -\frac{5}{2h^2}, aa \rightarrow -\frac{1}{12h^2}, bb \rightarrow \frac{4}{3h^2}, dd \rightarrow \frac{4}{3h^2}, ee \rightarrow -\frac{1}{12h^2} \right\} \right\}$$

$$aa = ee \rightarrow -1/12$$

$$bb = dd \rightarrow 4/3 \rightarrow 16/12$$

$$cc \rightarrow 5/2 \rightarrow -30/12$$

(* Quoficients per a la derivada central a tres punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi $y = a + b x + c x^2$ *)

$$\text{Solve}[\{(aa + bb + cc) == 0, \\ (aa - cc) == 0, \\ h^2(aa + cc) == 2\}, \{aa, bb, cc\}]$$

$$\left\{ \left\{ bb \rightarrow -\frac{2}{h^2}, aa \rightarrow \frac{1}{h^2}, cc \rightarrow \frac{1}{h^2} \right\} \right\}$$