Prove this theorem: *In one dimension there are no degenerate bound states.*

**Hint:**
Suppose there are two solutions, \( \psi_1 \) and \( \psi_2 \), with the same energy \( E \). Multiply the Schrödinger equation for \( \psi_1 \) by \( \psi_2 \), and the Schrödinger equation for \( \psi_2 \) by \( \psi_1 \), and subtract, to show that \((\psi_2 d\psi_1/dx - \psi_1 d\psi_2/dx)\) is a constant. Use the fact that for normalizable solutions \( \psi \to 0 \) at \( \pm \infty \) to demonstrate that this constant is in fact zero. Conclude that \( \psi_2 \) is a multiple of \( \psi_1 \), and hence that the two solutions are not distinct.

\[ -\frac{\hbar^2}{2m} \frac{d^2 \psi_1}{dx^2} + V\psi_1 = E\psi_1 \Rightarrow -\frac{\hbar^2}{2m} \frac{d^2 \psi_1}{dx^2} + V\psi_1 \psi_2 = E\psi_1 \psi_2 \]
\[ -\frac{\hbar^2}{2m} \frac{d^2 \psi_2}{dx^2} + V\psi_2 = E\psi_2 \Rightarrow -\frac{\hbar^2}{2m} \frac{d^2 \psi_2}{dx^2} + V\psi_1 \psi_2 = E\psi_1 \psi_2 \]

But \( \frac{d}{dx} \left[ \psi_2 \frac{d\psi_1}{dx} - \psi_1 \frac{d\psi_2}{dx} \right] = \psi_2 \frac{d^2 \psi_1}{dx^2} + \psi_1 \frac{d^2 \psi_2}{dx^2} - \psi_1 \frac{d\psi_1}{dx} \frac{d\psi_2}{dx} - \psi_1 \frac{d^2 \psi_2}{dx^2} = \psi_2 \frac{d^2 \psi_1}{dx^2} - \psi_1 \frac{d^2 \psi_2}{dx^2} \)

Since this is zero, it follows that \( \psi_2 \frac{d\psi_1}{dx} - \psi_1 \frac{d\psi_2}{dx} = K \) (a constant).

But \( \psi \to 0 \) at \( \infty \) so the constant must be zero. Thus \( \psi_2 \frac{d\psi_1}{dx} = \psi_1 \frac{d\psi_2}{dx} \), or \( \frac{1}{\psi_1} \frac{d\psi_1}{dx} = \frac{1}{\psi_2} \frac{d\psi_2}{dx} \),

so \( \ln \psi_1 = \ln \psi_2 + \text{constant} \), or \( \psi_1 = (\text{constant})\psi_2 \).