Aharonov-Bohm Effect for pedestrian

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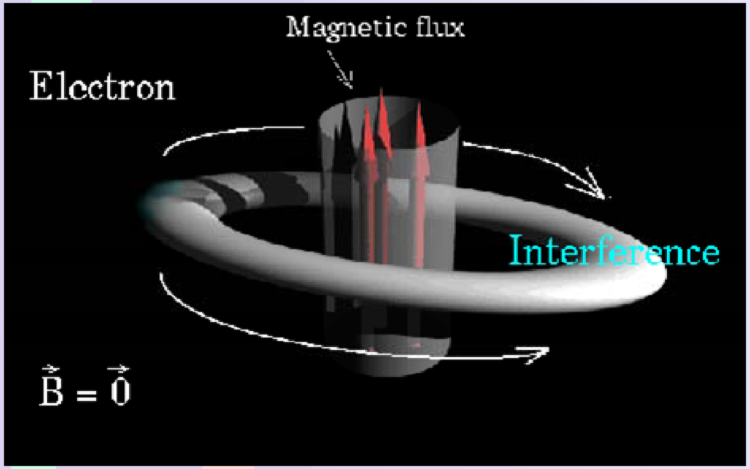
Castelló (SPAIN), June 2005

Aharonov-Bohm Effect

Y. Aharonov, D. Bohm, Phys. Rev. ***115*** *(1959) 485*



- Classical mechanics: equations of motion can always be expressed in term of field alone.

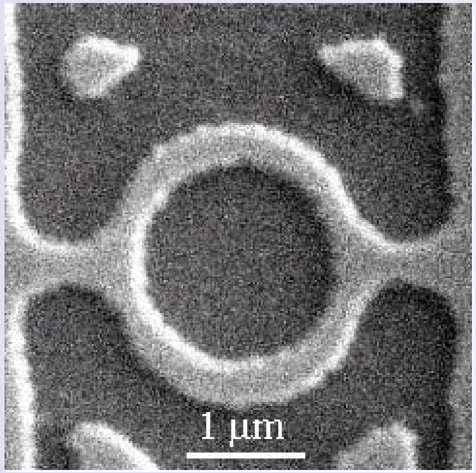
- Quantum mechanics: canonical formalism. Potentials cannot be eliminated.

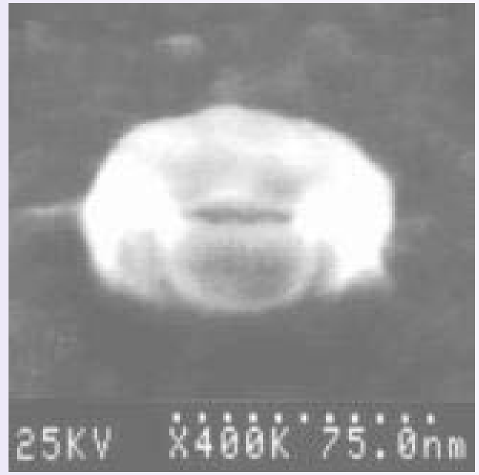
- An electron can be influenced by the potentials even if no fields acts upon it.

- $\oint A dl$ Gauge independent

Semiconductor Quantum Rings

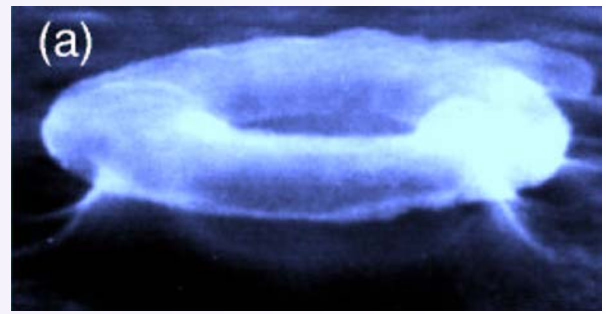
Litographic rings
GaAs/AlGaAs



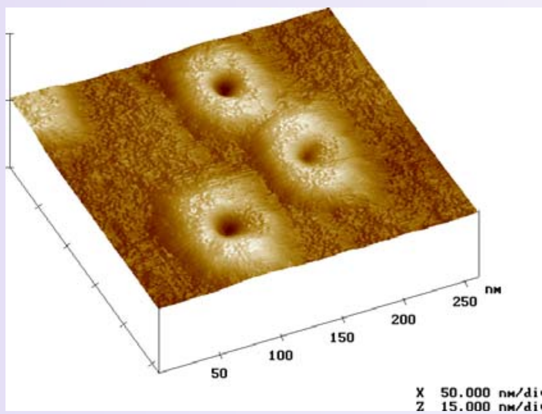


A.Fuhrer et al., Nature ***413*** *(2001) 822;*

M. Bayer et al., Phys. Rev. Lett. ***90*** *(2003) 186801.*



self-assembled rings
InAs

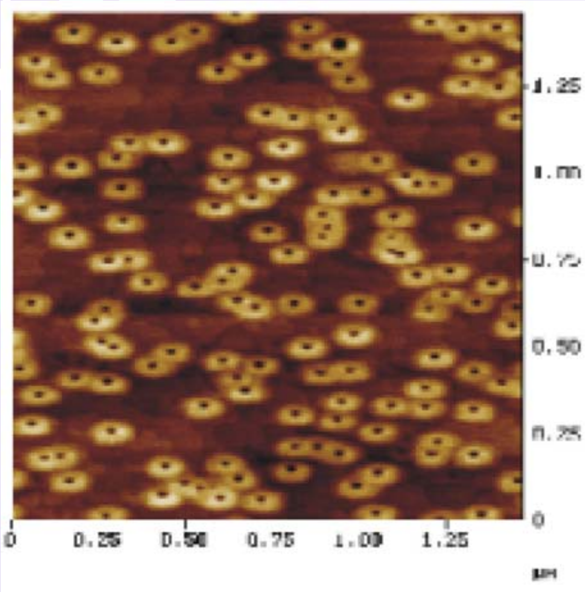


J.M.García et al., Appl. Phys. Lett. ***71*** *(1997) 2014*

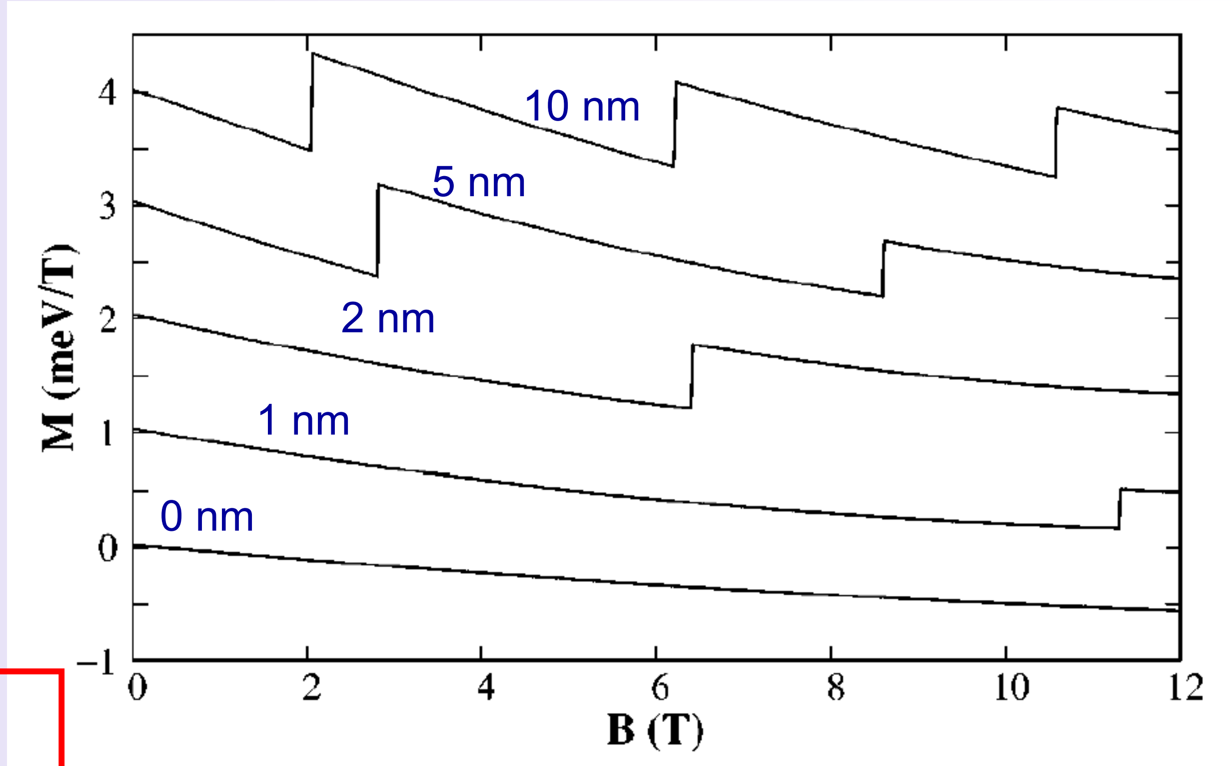
T. Raz et al., Appl. Phys. Lett ***82*** *(2003) 1706*

B.C. Lee, C.P. Lee, Nanotech. ***15*** *(2004) 848*

Magnetization: finger print



R_{ext} = 60 nm

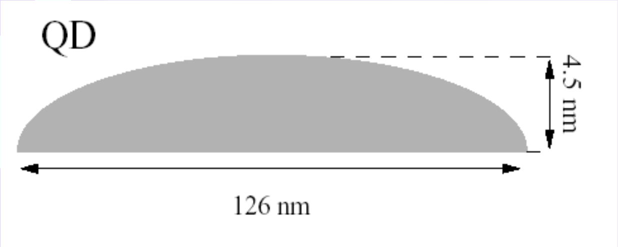


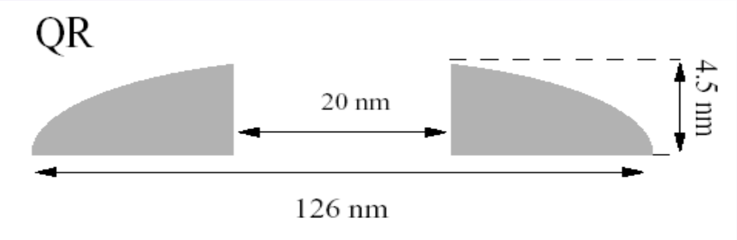
Is the ring-like geometry preserved after the sample is covered with matrix material?

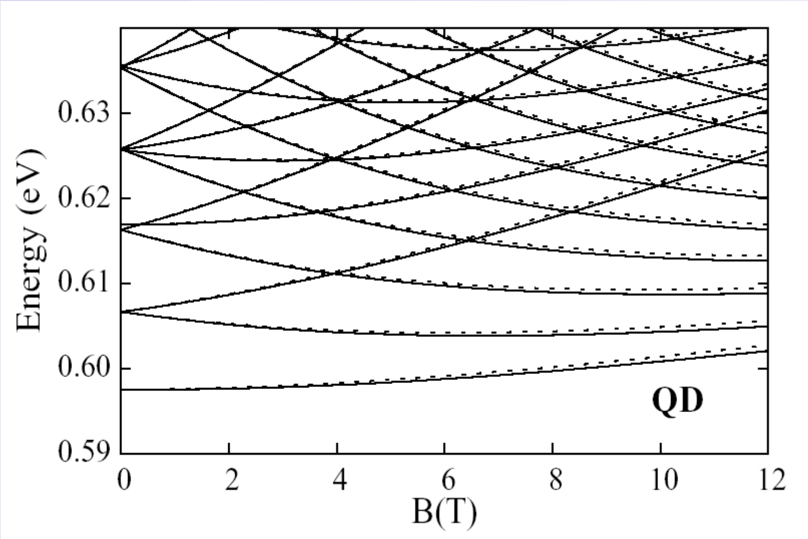
One electron magnetization (T=0 K)

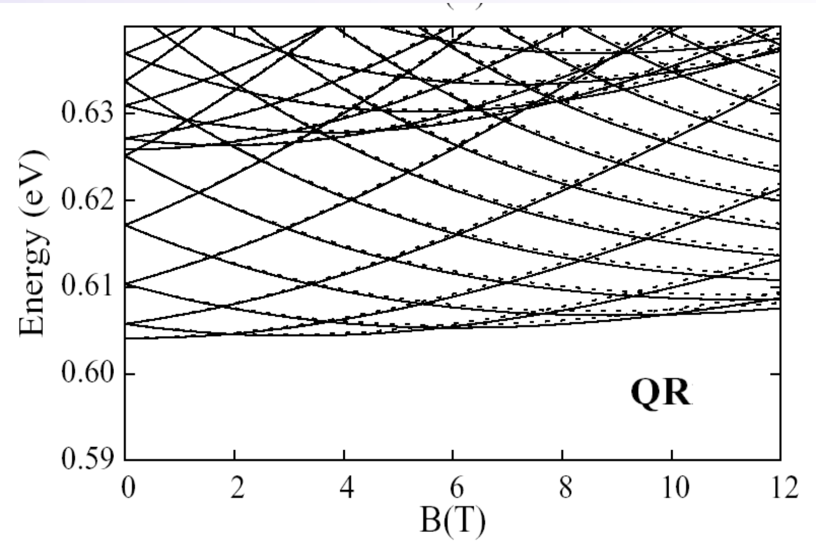
*J.I. Climente J.Planelles and J.L.Movilla Phys. Rev. B **70** (2004) 081301(R)*

Energy vs. magnetic field of one electron in a QD and a QR



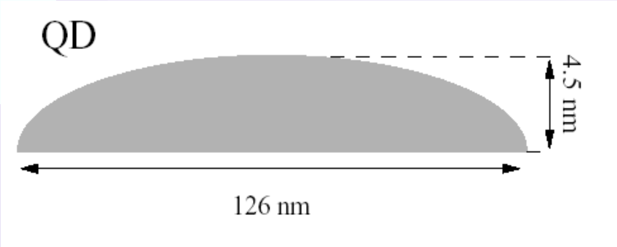


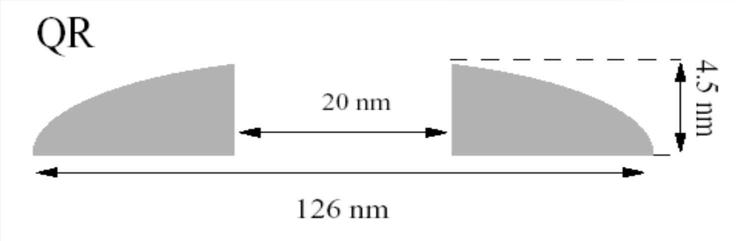


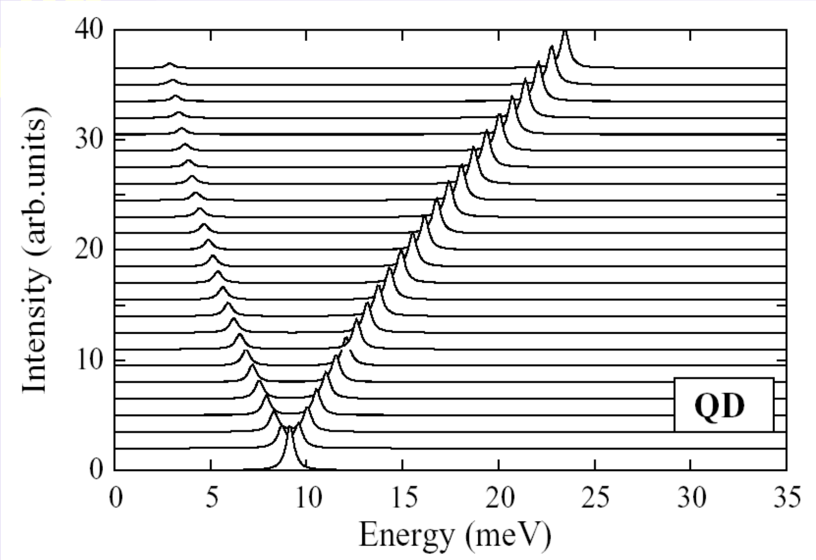


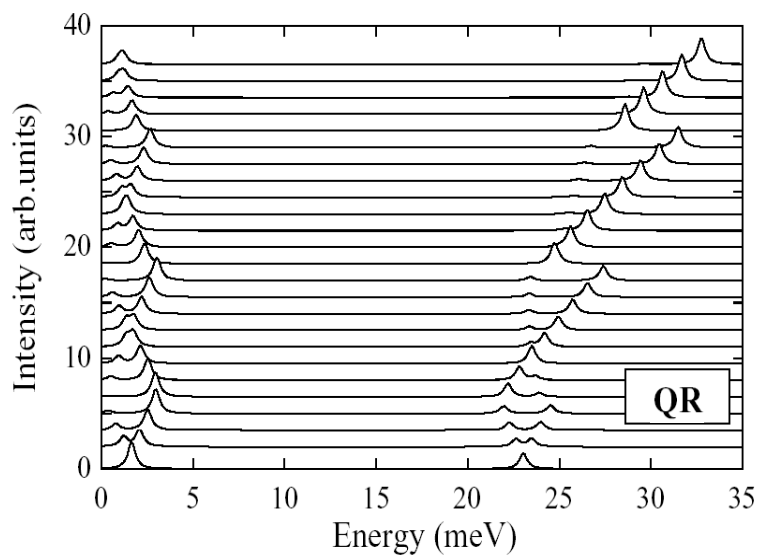
J. Planelles y J.I. Climente, arXiv: cond-mat/0412552;

FIR absorption of one electron in QD and QR









J. Planelles y J.I. Climente, arXiv: cond-mat/0412552;

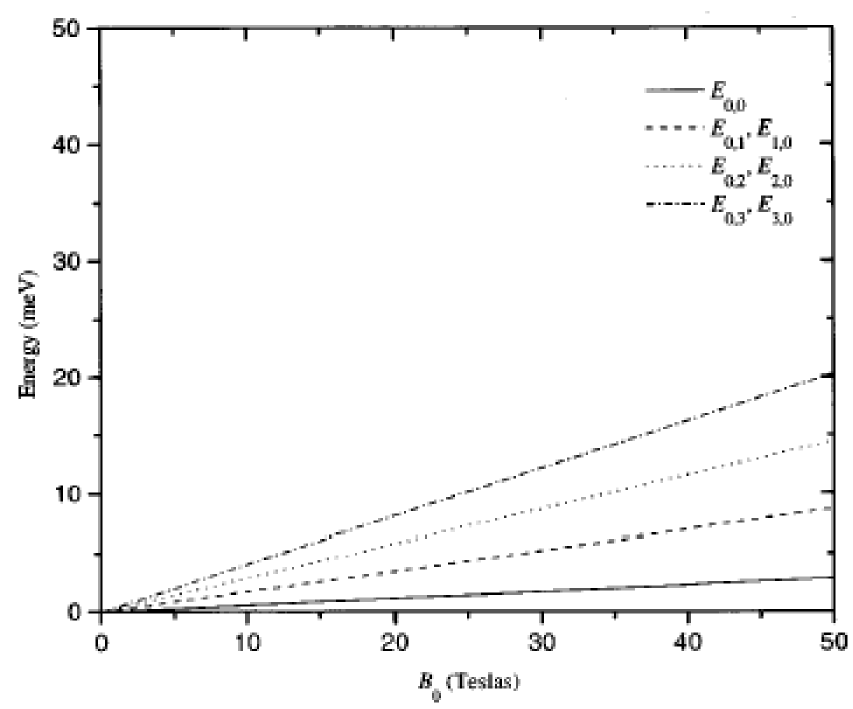
Electron in a magnetic field

$$\hat{\mathcal{H}} = \frac{(\hat{p} - eA)^2}{2m_e} + V$$

$$\vec{B} = B_0\vec{k}$$

$$\vec{A} = (-\tfrac{1}{2}y\,B_0, \tfrac{1}{2}x\,B_0, 0)$$

$$\begin{aligned}\hat{\mathcal{H}} &= -\frac{\hbar^2}{2m_e}\nabla^2 - \frac{eB}{2m_e}\hat{L}_z + \frac{e^2B^2}{8m_e}\rho^2 + V \\ &= \frac{\hat{p}_z^2}{2m_e} + \hat{\mathcal{H}}_{HO}^{2D} - \frac{eB}{2m_e}\hat{L}_z + V\end{aligned}$$

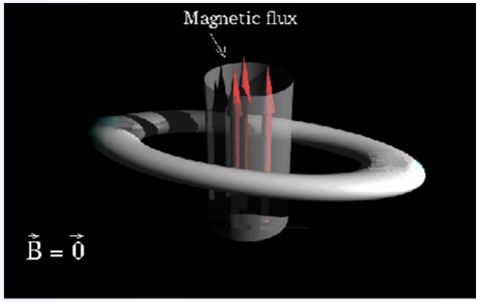


- Landau levels
- No crossings

Electron in a potential vector but no magnetic field

$B = B_0$ if $0 < \rho < a$ and $B = 0$ otherwise

$$\vec{A} = A_\phi \vec{u}_\phi = \begin{cases} \frac{1}{2} B \rho \vec{u}_\phi & 0 < \rho < a \\ \frac{Ba^2}{2\rho} \vec{u}_\phi & a < \rho < \infty \end{cases}$$



$$\hat{\mathcal{H}} = \frac{(\hat{p} - eA)^2}{2m_e} + V$$

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2m_e}\nabla^2 + \frac{i\hbar e}{m_e}\frac{Ba^2}{2\rho^2}\frac{\partial}{\partial\phi} + \frac{e^2B^2a^4}{8m_e\rho^2} + V$$

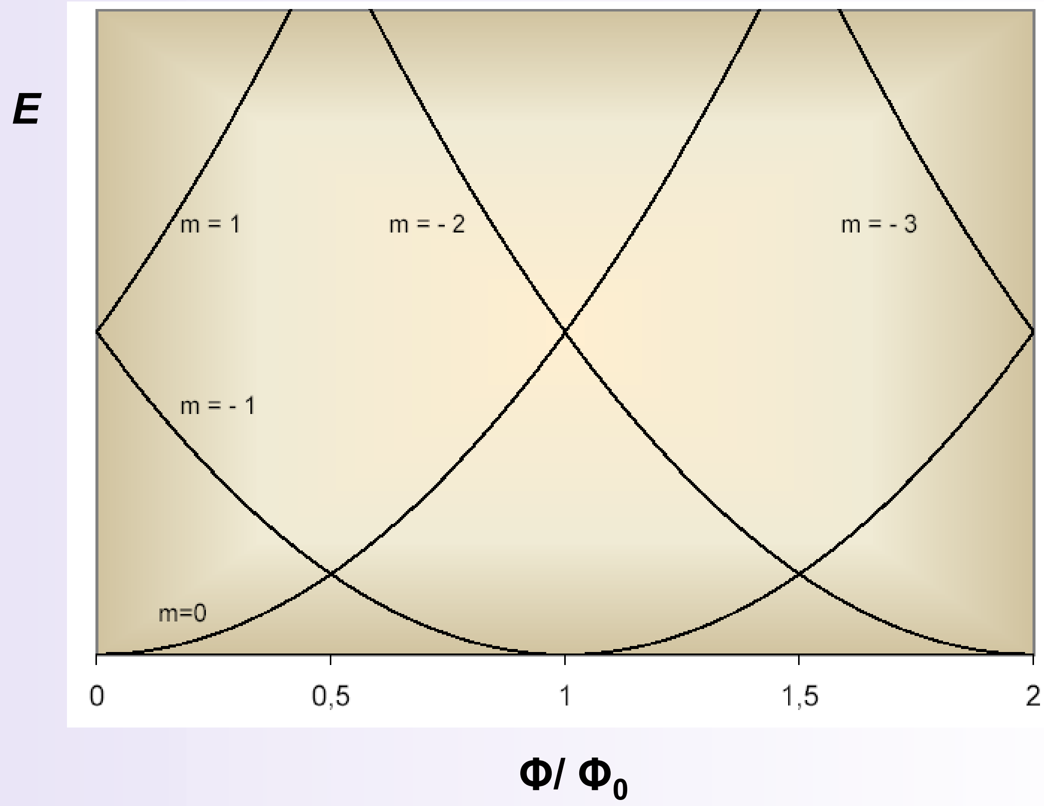
$$\frac{\partial}{\partial\phi} \to \frac{\partial}{\partial\phi} - \frac{i\,eB\,a^2}{2\,\hbar} = \frac{\partial}{\partial\phi} + i\frac{\Phi}{\Phi_0}$$

Aharonov-Bohm Effect

1D QR

$$\hat{\mathcal{H}} = -\frac{\hbar^2}{2m_e R^2}\left(\frac{\partial}{\partial\phi} + i\frac{\Phi}{\Phi_0}\right)^2$$

$$E_m = \frac{1}{2}(m+F)^2$$



- Periodic symmetry changes of the energy levels

- Energetic oscillations

Fractional Aharonov-Bohm Effect

$$\hat{\mathcal{H}}(1,2) = -\frac{1}{2R^2}\left(\frac{\partial}{\partial\phi_1} + iF\right)^2 - \frac{1}{2R^2}\left(\frac{\partial}{\partial\phi_2} + iF\right)^2 + \frac{1}{r_{12}}$$

$$r_{12} = 2\,R|\sin\tfrac{\phi_2-\phi_1}{2}|$$

Disregarding the Coulomb interaction:

$$E(m_1, m_2) = \frac{1}{2R^2}\left[(m_1 + F)^2 + (m_2 + F)^2\right]$$

→ periodic changes of the ground state at the same values of flux as in the one electron case.

Eigenfuctions: singlets (S) and triplets (T)

$$|m_1, m_2; S/T\rangle = e^{im_1\phi_1}e^{im_2\phi_2} \pm e^{im_1\phi_2}e^{im_2\phi_1}$$

$|m, m; T\rangle$ does not exist (is zero) → ground state always singlet.

Fractional Aharonov-Bohm Effect

$$s = \frac{1}{2}(\phi_1 + \phi_2)$$

$$r = \frac{1}{2}(\phi_1 - \phi_2)$$

$$|M, m; S\rangle = e^{iMs} \cos mr$$

$$|M, m; T\rangle = e^{iMs} \sin mr$$

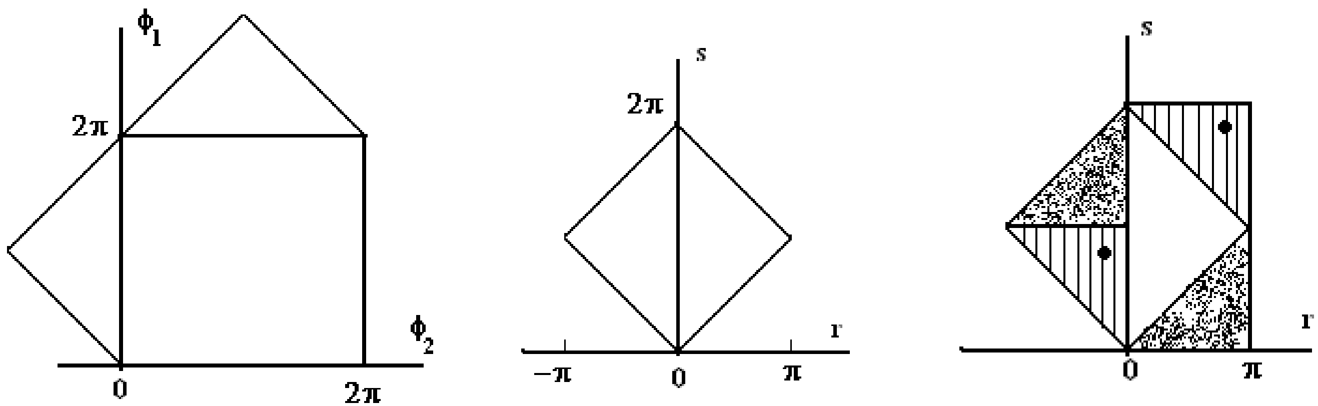
no triplets for m=0

$$M = m_1 + m_2, \; m = m_1 - m_2$$

M, m same parity

$$E(M, m) = \frac{1}{4R^2}\left[(M + 2F)^2 + m^2\right]$$

Mapping between (ϕ_1,ϕ_2) and (r,s) domains.



$$\phi_1 \equiv \phi_1 + 2\pi$$

$$\Downarrow$$

$$(s, r) \equiv (s + \pi, r + \pi)$$

Fractional Aharonov-Bohm Effect

$$\hat{\mathcal{H}}(1,2) = -\frac{1}{4R^2}\left(\frac{\partial}{\partial s} + 2\,iF\right)^2 - \frac{1}{4R^2}\frac{\partial^2}{\partial r^2} = \hat{\mathcal{H}}_s + \hat{\mathcal{H}}_r$$

domain

$$0 < s < 2\pi$$

$$0 < r < \pi$$

With Coulomb interaction: $$\hat{\mathcal{H}}_r = -\frac{1}{4R^2}\frac{\partial^2}{\partial r^2} + \frac{1}{2\,R|\sin r|}$$

Eigenfuctions: $e^{i\,M\,s}\Psi_n(r)$

Pauli's principle restrictions in the presence of Coulomb interactions:

$$(s,r) \equiv (s+\pi, r+\pi)$$

$$\mathcal{P}_{12}s = s$$

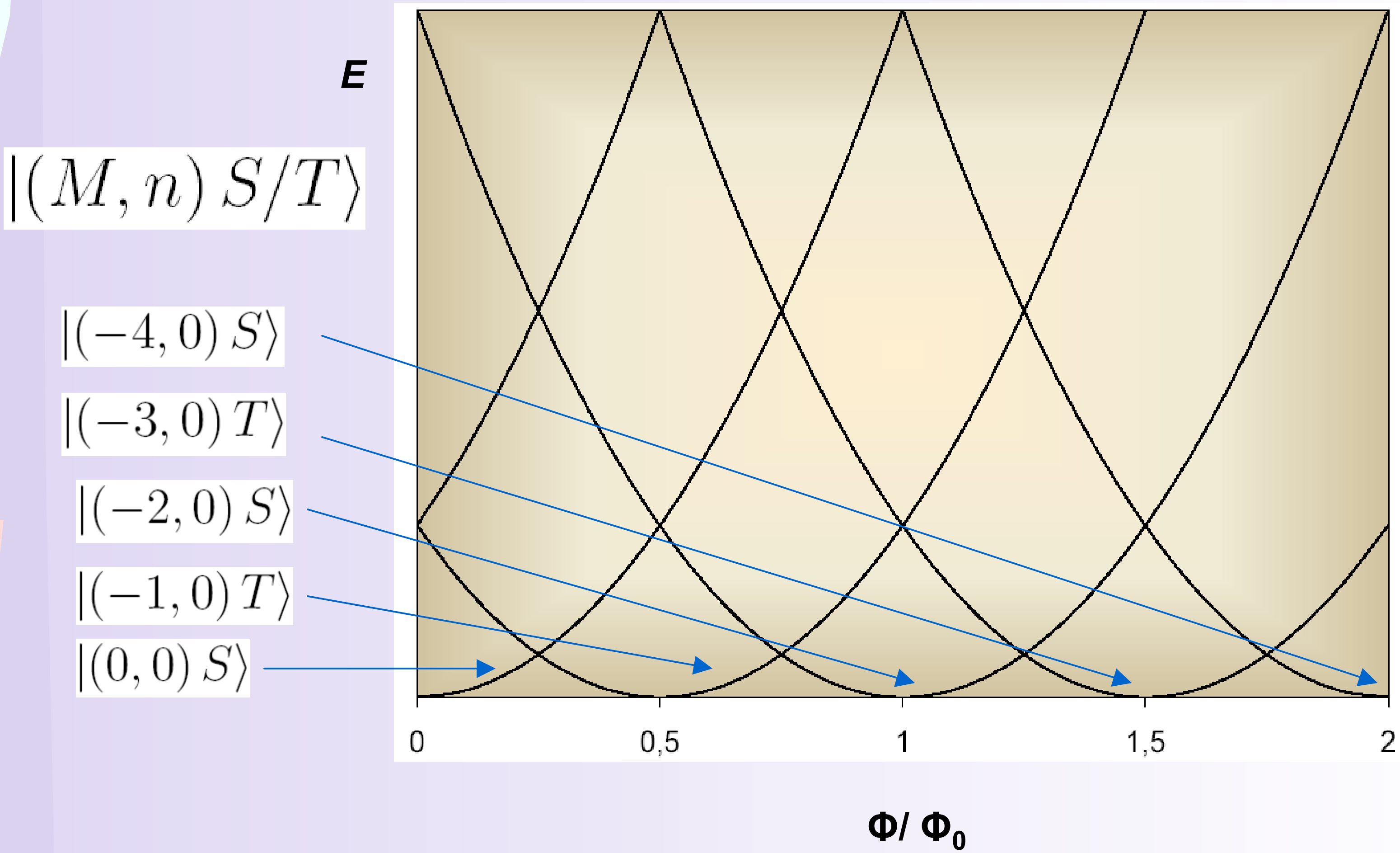
$$\mathcal{P}_{12}r = -r$$

$$e^{i\,M\,s}\Psi_n(-r) = e^{i\,M\,s}e^{i\,M\,\pi}\Psi_n(\pi - r)$$

$$\hat{\mathcal{P}}_{12}\Psi_n(r) = (-1)^M\Psi_n(\pi - r)$$

$$\hat{\mathcal{P}}_{12}\Psi_n(r) = (-1)^{(M+n)}\Psi_n(r)$$

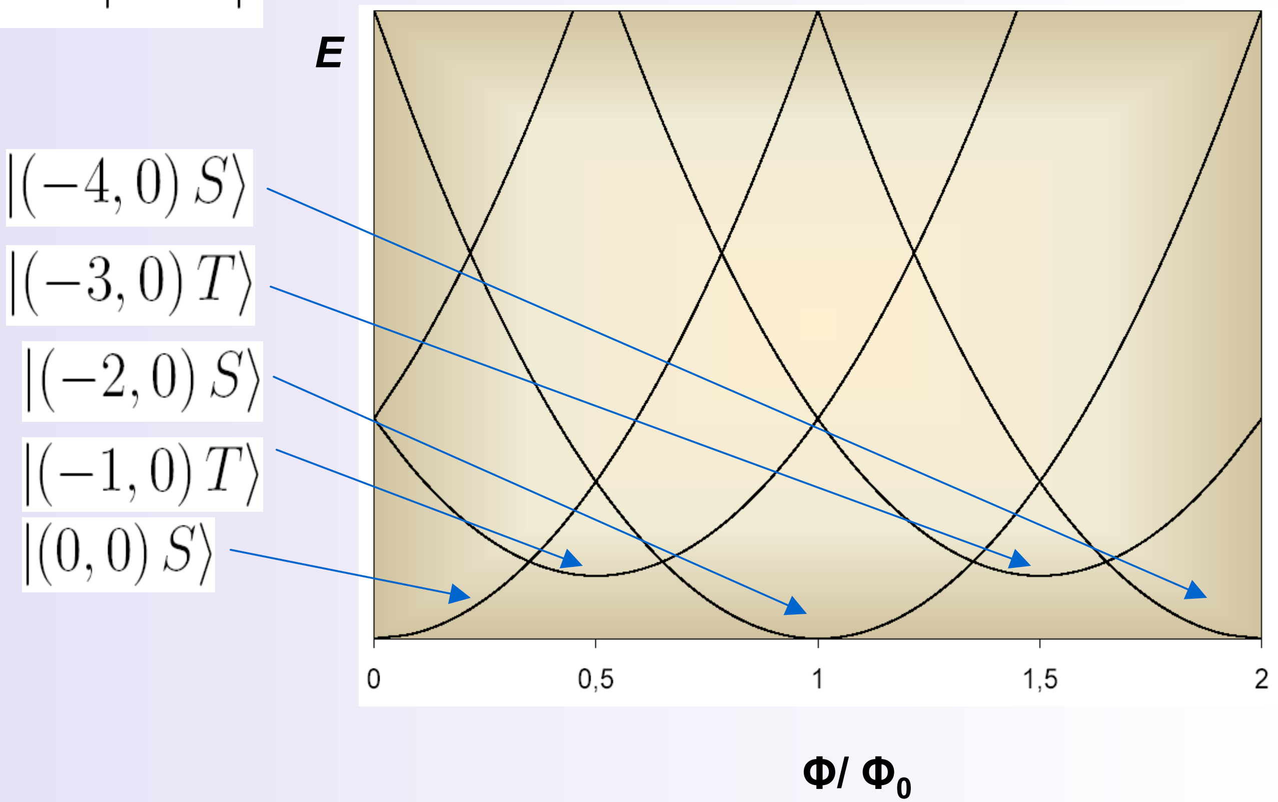
Fractional Aharonov-Bohm Effect



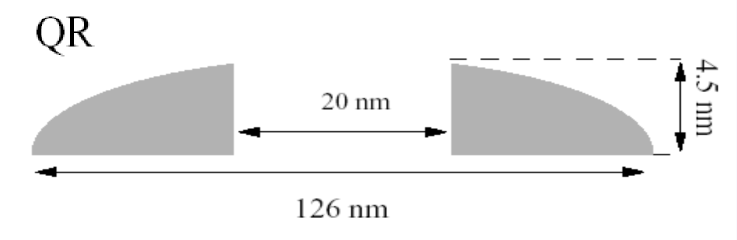
Fractional Aharonov-Bohm Effect

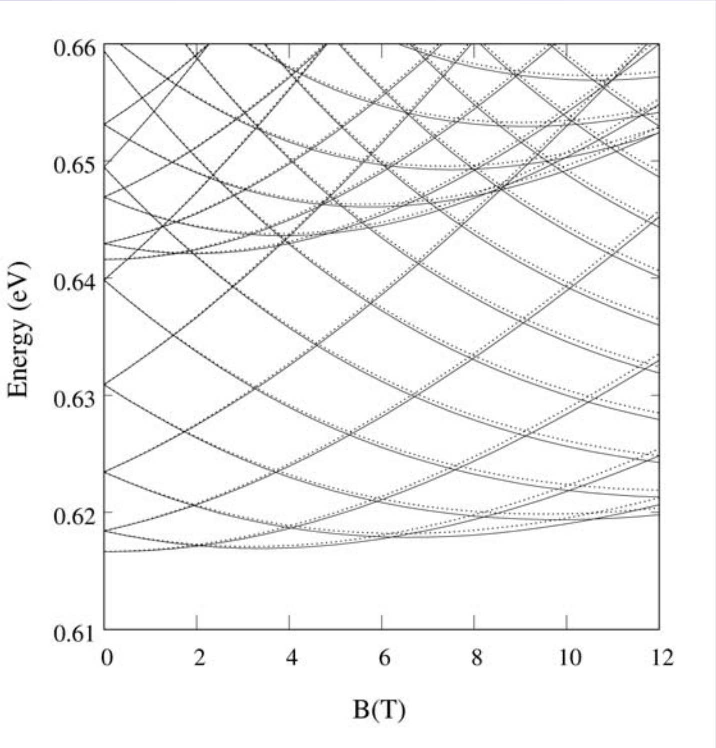
Coulomb interaction in a 1D system is unrealistically large.

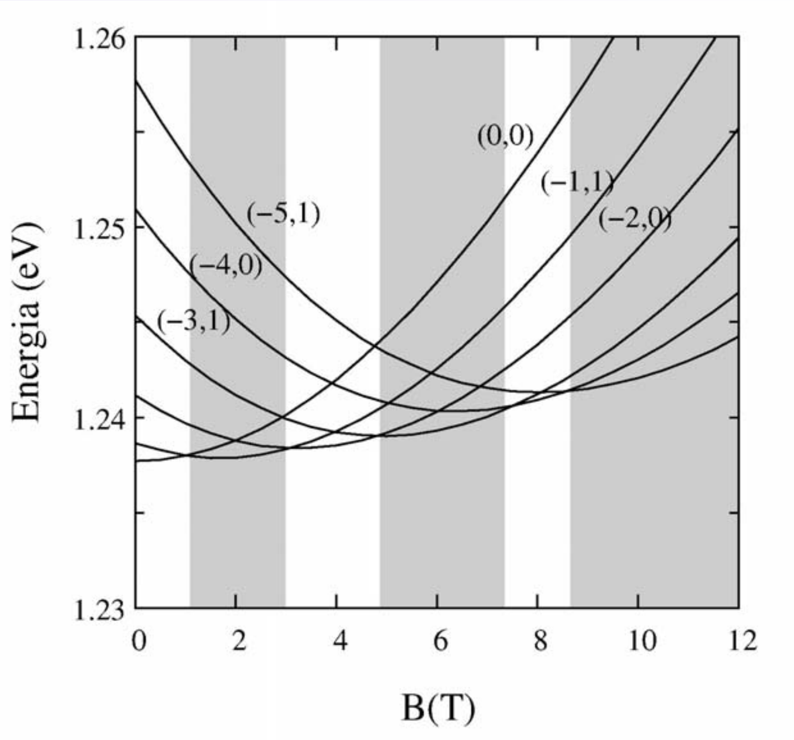
$$\mathcal{H}_\xi = -\frac{1}{4R^2}\frac{\partial^2}{\partial r^2} + \frac{1}{\xi + 2\,R\,|\sin r|}$$



Fractional Aharonov-Bohm Effect





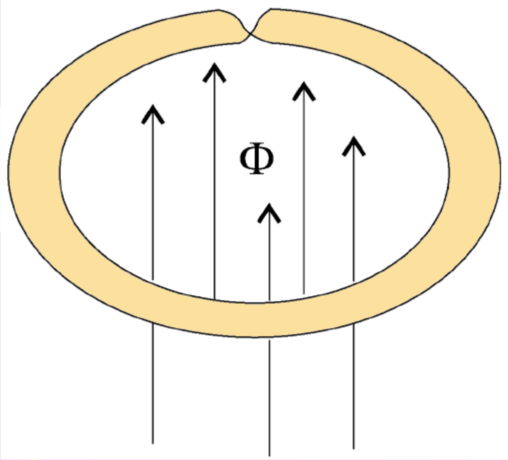


1 electron

2 electrons coulomb interaction

J.I. Climente, J. Planelles and F. Rajadell, J. Phys. Condens. Matter ***17*** *(2005) 1573*

Aharonov-Bohm Effect in a Möbius strip



Microscopic Semiconductor $NbSe_3$ Moebius strip
S. Tanda et al., Nature ***417*** *(2002) 397.*

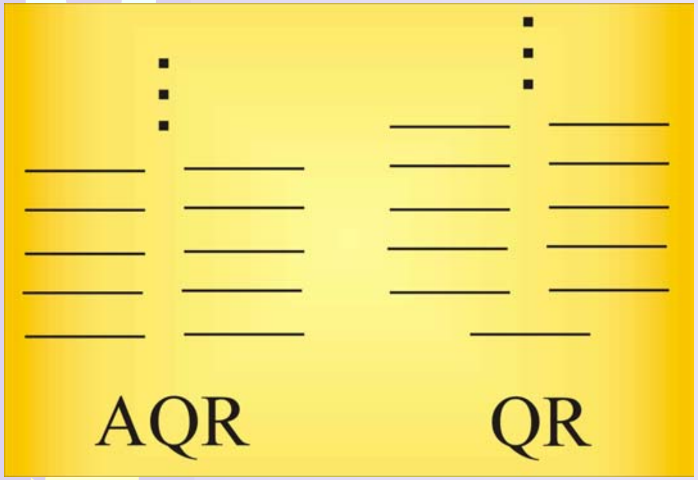
Möbius strip cannot be pressed into a 1D structure

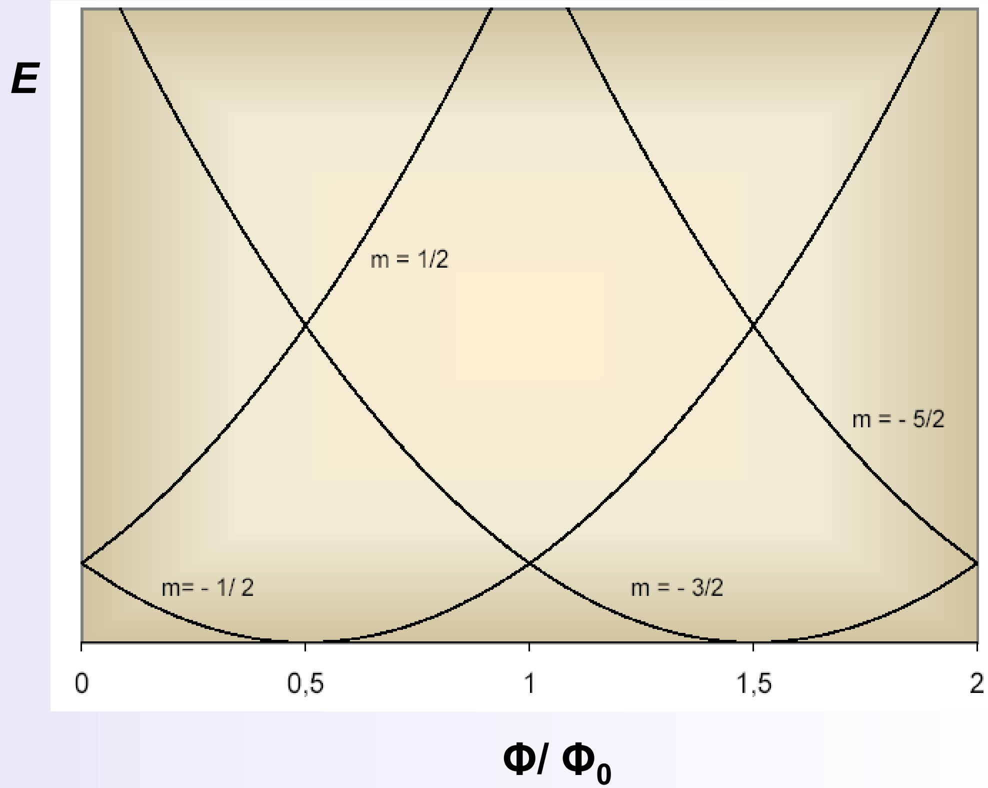
→1D QR $\hat{\mathcal{H}} = -\frac{\hbar^2}{2m_e R^2}\left(\frac{\partial}{\partial\phi} + i\frac{\Phi}{\Phi_0}\right)^2$ → $E_m = \frac{1}{2}(m+F)^2$

with antiperiodic BCs $\Psi_m(\phi + 2\pi) = -\Psi_m(\phi)$

→ $m = \pm 1/2, \pm 3/2, \pm 5/2, \ldots$

Aharonov-Bohm Effect in a Möbius strip

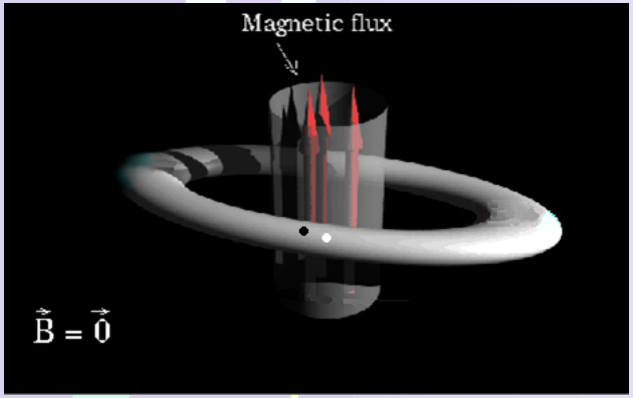




$$\hat{\mathcal{P}}_{12}\Psi_n(r) = (-1)^{(M+n+1)}\Psi_n(r)$$

AQR same picture as in QR except for a shift of half flux unit.

Optic Aharonov-Bohm effect: excitons



Exciton is a neutral entity.... Should it be sensitive to the applied magnetic flux?

$$\hat{\mathcal{H}} = -\frac{1}{2m_e^* R^2}\left(\frac{\partial}{\partial \phi_e} + iF\right)^2 - \frac{1}{2m_h^* R^2}\left(\frac{\partial}{\partial \phi_h} - i\,F\right)^2 - \frac{1}{2R|\sin\frac{\phi_e - \phi_h}{2}|}$$

Without Coulomb term:

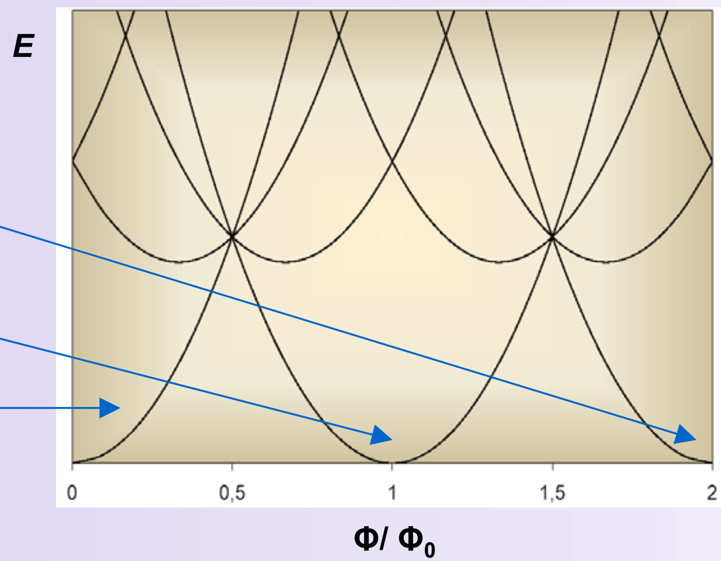
$$E - E_g = \frac{1}{2m_e^* R^2}(M_e + F)^2 + \frac{1}{2m_h^* R^2}(M_h - F)^2$$

$|M_e, M_h\rangle$

$|-2, 2\rangle$

$|-1, 1\rangle$

$|0, 0\rangle$

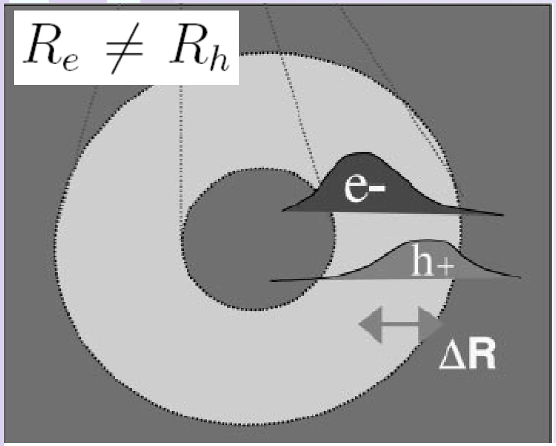


$$M_L = M_e + M_h = 0$$

⬇

Exciton always Bright

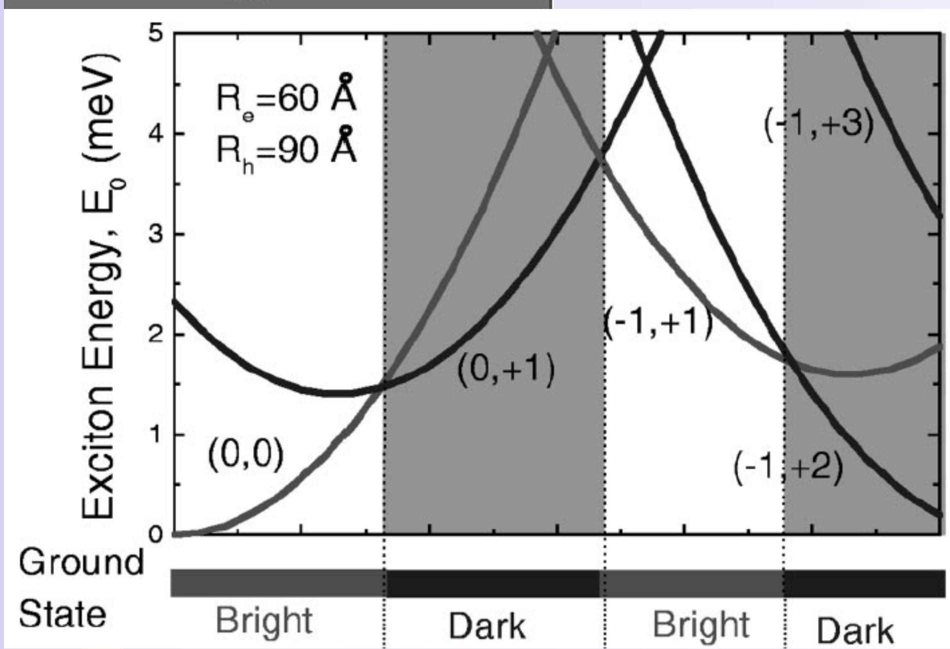
Optic Aharonov-Bohm effect

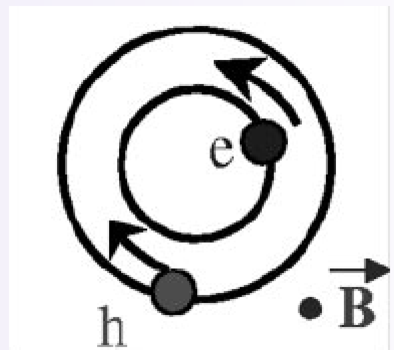


A.O. Govorov et al. Phys. Rev. B ***68*** *(2003) 075307*

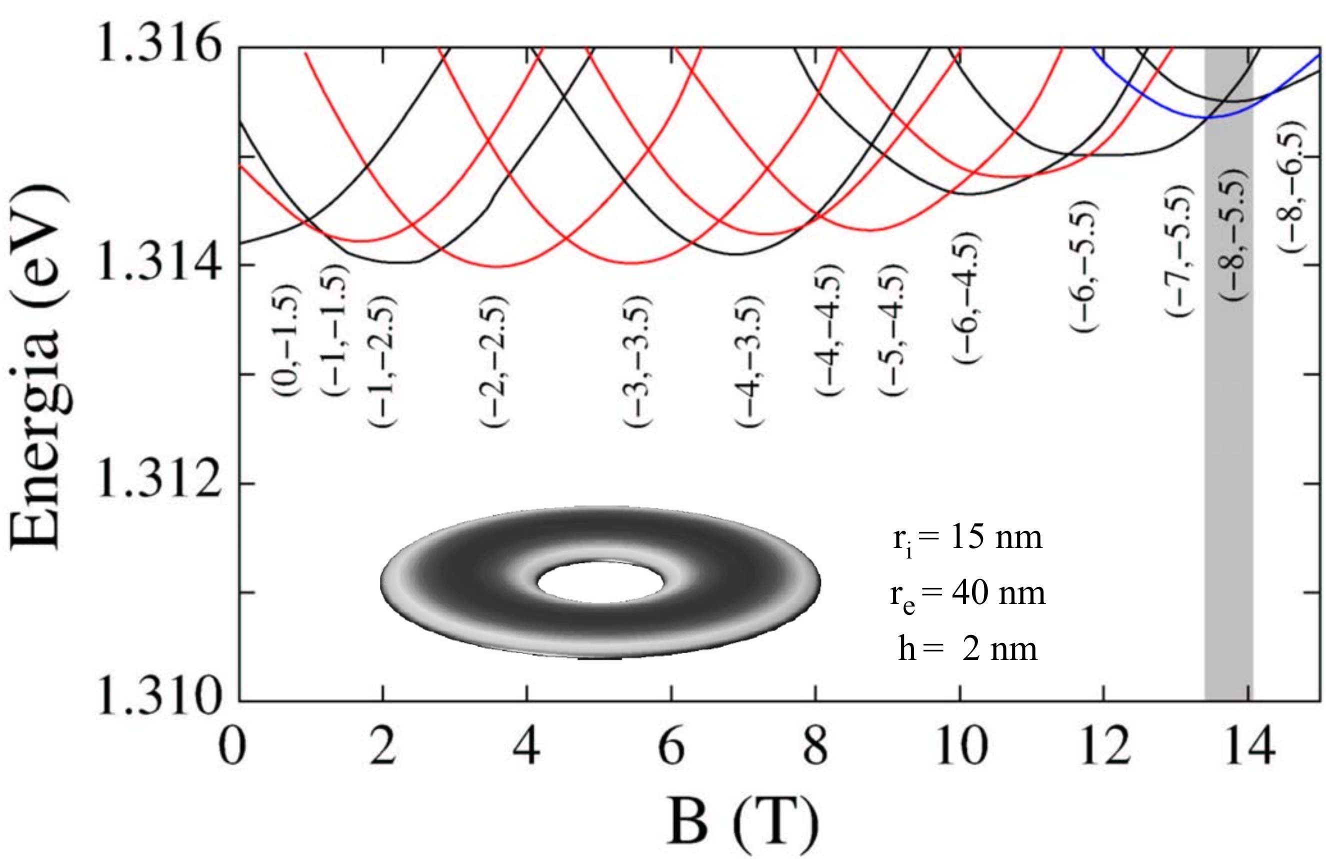
Without Coulomb term:

$$E = E_g + \frac{1}{2m_e^* R_e^2}(M_e + F_e)^2 + \frac{1}{2m_h^* R_h^2}(M_h - F_h)^2$$





Optic Aharonov-Bohm effect

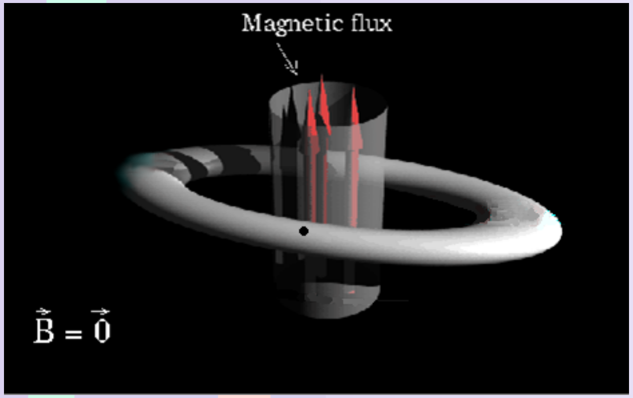


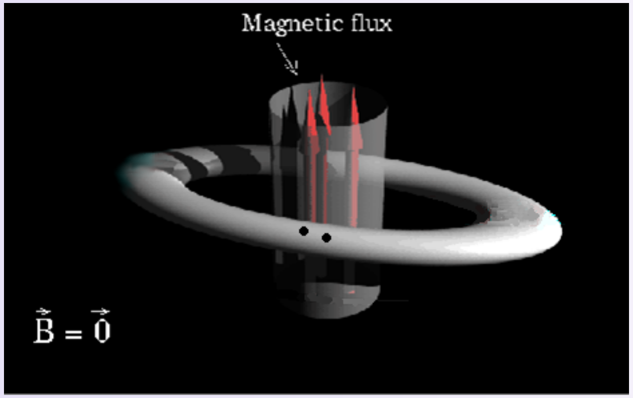
*J.I. Climente, J. Planelles and W. Jaskólski, Phys. Rev. B **68** (2003) 075307*

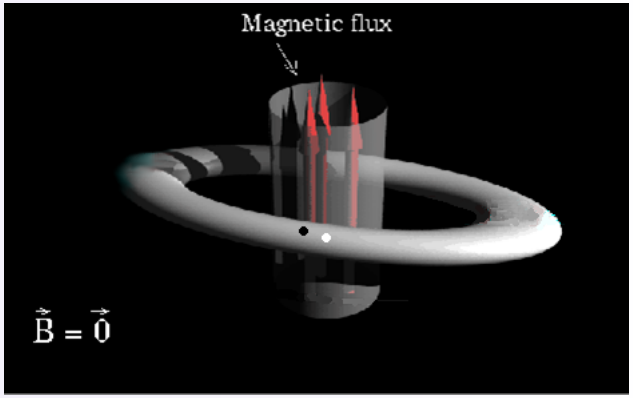
Experimentally no trace of this effect was detected for magnetic-field values up to 9 T (D. Haft et. al, Physica E **13**, (2002)165)

Aharonov-Bohm Effect

Observable pure quantum mechanical effect: Periodic oscillations







AB effect
period 1F

Fractional AB
period ½ F

Optic AB
Only if $\Delta F_{e,h} \neq 0$