

# Initial modelling strategies for geometrical reconstruction optimization-based approaches

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## Abstract

A general-purpose and automatic (or, at least, easy-to-use) system (called REFER) to create 3D geometrical models from 2D sketches, through a geometrical reconstruction process, is our present objective.

In this paper, some key aspects of the heuristic strategies proposed by the authors to generate initial «tentative» models to avoid trivial minimums, and convey the optimization processes towards the psychologically plausible solution, are discussed.

**Keywords:** Image processing, geometrical reconstruction, optimization.

## 1. INTRODUCTION

Despite the powerful CAD systems nowadays present in the market, many engineers and designers still prefer pen and pencil, mainly in the more conceptual steps of design process, in which only an incomplete set of requirements and abstract ideas about the design are known. This is because CAD systems are too rigid to allow a fast scan of incomplete and non-formalized ideas and models. In other words, manual sketches still are the "natural" language engineers and designers use to synthesize new designs. Moreover, computer are not only unable to help designers to integrate «sketching methodologies» in a more complex «ideation machine», but also are «blind» to see the final sketch. This deficiency forces draftsmen to fully reconstruct the models in a CAD system once the sketching process ends. Draftsmen read the final sketch and guide the CAD system to construct the corresponding models.

«Artificial Vision» or «Computer Vision» is considered to be the field where all the tools and strategies employed to make the computers to «see» (that is, to make sense on all the images they can input through different devices) is developed. In fact, Artificial Vision can be divided into two parts; where the first one is more concerned about 3D scenes, and the second one deals with 2D images (see figure 1).

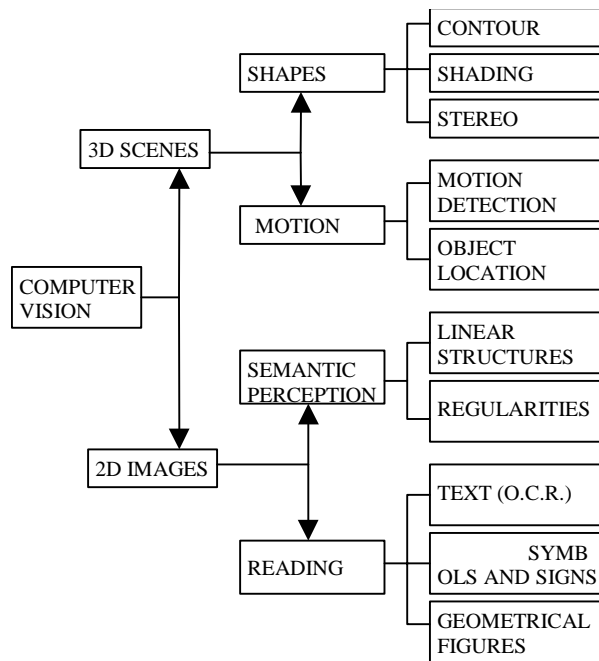


Figure 1. Fields of interest in Computer Vision.

When 3D scenes are considered, shapes determination and motion detection are the usual objectives. Here, people is faced to the challenging problem of helping robots to interact with its surrounding and real world by identifying which objects are present in the scene and how they move (or where and how are they placed). It may be noticed that, in general, identification of the objects does not include a detailed description about its geometry. In other words, the problem is to determine which is the object, not to know its exact geometry. Moreover, the strategy to capture the geometry of all objects present in a scene where a robot moves would be very inefficient, because, robots are expected to act as humans do, getting the minimum information needed to avoid crashes

with objects, without knowing its geometry if full detail.

When 2D images constitute the input we can, again, divide the problem in two different aspects. First, the one related to some phenomenon we want to study and concerned on how to make sense from images obtained through all kind of cameras, scanners and sensors. Here, the objective is filtering the obtained images to describe them in a compact or convenient form, or to highlight some pieces of information presents on it. Second, there are some researchers trying to add the capabilities the computers need to «read» information in the way humans can do it. Character recovery, interpretation of symbols and signs, and, finally, making computer able to interpret 2D images in terms of 3D model projections. This last problem is named «reconstruction» (or more precisely «geometrical reconstruction») and implies determination of geometrical and topological relations of all atomic parts of one object.

## 2. GEOMETRICAL RECONSTRUCTION.

The field concerned on automatic recovering of the geometrical 3D models implicitly contained in 2D geometric figures is known as Geometrical Reconstruction. Its origins can be traced with a reduced set of references [1], [2], [3]. And the present situation can be summarized pointing out that most of the known reconstruction approaches are in experimental stages, and can interpret (with few errors), all kind of polyhedral objects. Interpretation of the most usual 3D primitives (like cylinders, spheres, etc.) is also considered by some of the approaches. Anyway, when object complexity increases, automatic processes usually give pass to semiautomatic ones.

Geometrical Reconstruction cannot be seen as an exclusive problem of people working in the area of Computer Graphics. Because, as it has been stated many times, «CAD systems based on solid modeling do not have the right interface» [4], and the final goal is getting an «ideation machine» to help designers in the initial, and more conceptual, steps of design process [5]. In other words, computers must interact with designers in a friendly way and during the whole design cycle. Consequently, one graphical language must be defined (or adopted, in case of one already existing language been chosen) to improve the present communication between designers and CAD systems. This means that CAD systems must «see» in the way designers do. Today, CAD systems have non-sequential (graphic) outputs, but accept only sequential (verbal) inputs. This is a direct consequence of current state of evolution in Computer Graphics, constrained by the sequential nature of algorithmic languages used for programming tasks. Yet it is important to notice that we do not claim for the physical implementation to become non-sequential; it is the conceptual model (and the interaction front end, which is the same to say the «language») the one which must be graphical. Obviously, to achieve this goal some pure computer world and some pure Engineering Graphics aspects must be integrated.

In this context, it must be pointed out that, from the point of view of Geometry, it has been always very well known that full recovery of a geometrical 3D model from one single projection of it is not possible. Nevertheless, from the field of psychologists it is also very well known the fact that humans seem to have no problems to identify 3D models depicted in 2D images. What is more, it seems to have a great consensus about which is the «correct» and «single» model all humans see in every picture. The reason comes from the fact that humans, when «reading» drawings, do implicit recovery actions. According to Gestalt school, this is because human perception holds some common characteristics, called «principles of organization». Consequently, pure Visual Perception rules must also be considered to face the reconstruction problem.

### **3. OPTIMIZATION APPROACH**

Visual Perception is assumed to act in a tentative and iterative way. In other words, some «organization forces» guide the brain to interpret the figure in terms of Gestalt rules («good form» laws). Successive organizational attempts are carried out, until the organization forces are minimized. When contradictory interpretation appear, the brain looks for the best organized form, and discards those forces been incompatible with the prevailing form.

Consequently, an iterative process where some initial solution is refined according with some perceived characteristics, seems to be a good strategy to get what the sole application of geometry rules cannot obtain: a 3D psychologically plausible model. This process imitates the perception strategy, and discards those initial assumptions they are proved to be no valid. This is why Reconstruction can be described in terms of a mathematical optimization problem.

Obviously the configuration space for the optimization problem, depends on the nature the objects to be reconstructed have. When polyhedral objects are considered, the points whose z-coordinates are calculated are those junctions in the departure graph that are candidate to correspond to 3D vertices in the model. The formulation of such strategy departs from the following key ideas, proposed by Marill [6]: a) (X,Y) coordinates of every vertex in the graph are equal to (X,Y) coordinates of the corresponding vertex in the model, b) edges in the model have a univocal correspondence to line segments in the graph, and connect the same vertices than line segments do. In a more general context, control points of some curves

in the departure graph must be used as candidate to correspond to control points in some 3D curves or surfaces in the model.

Thus, the problem can be described as «Minimize  $F(z)$ », where:

- $z = (z_1, z_2, \dots, z_n)$  is the set of  $z$ -coordinates that constitute the independent variables vector.
- $n$  is the number of vertices or «control points» that fully determine the shape of the model, and determines the «order» or the problem.

The consequence of  $Z$  coordinates to be the variables is an infinite number of geometrically valid models, because they all can be projected in the same figure. Marill called «orthographic extension» to the full set of infinite three-dimensional objects whose orthogonal projection is equal to the given graph. Some of the model belonging to the orthographic extension of a given 2D graph are shown in figure 2.

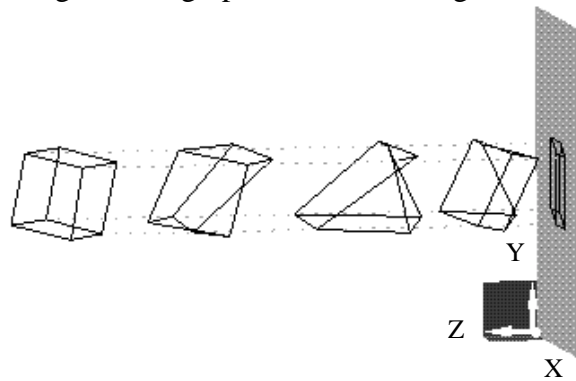


Figure 2. Orthographic extension of a 2D graph.

The second key idea in the proposed approach is to develop some criteria to choose one particular model among the infinite ones contained in the Extension. As was said before, there is, in general, only one «good» solution, in terms of psychological plausibility. That is, human observers seem to have no doubts to choose the appropriate solution among all those constituting the Orthographic Extension, as Leclerc and Fischler perceived [7].

«Regularities» are the way to make explicit such human perceptions. The foundations of this concept can be traced back in classic psychology reference books like the one by Koffka [8]. Nevertheless, the notion of regularity requires a more detailed explanation. In fact the first appearances of the word «regularities» associated with graphical representations are due to the Gestalt Psychologists, who named regularities to «those relations that cannot be an accident». Latter on, when some Scientific Visualization areas, related with semantic perception, began their own research, the term «regularities» became synonymous of «template to describe images in a compact or convenient form». Finally, in the geometrical reconstruction field, regularities are interpreted as «those properties of the image that must correspond to some properties the searched model also has» (See [1], [9] and [10]).

Consequently, it is supposed that properties describing how the model must be can be discovered by inspecting the departure image. Then, those regularities can be formulated in a mathematical language. Hence, the Objective Function can be easily defined in terms or regularities in the following form:

$$F = \sum \alpha_j R_j(z) \quad (1)$$

where,

- $\alpha_j$  is the  $j$ -th weighting coefficient, and
- $R_j(z)$  is the  $j$ -th regularity.

Regularities must be expressed in terms of the independent variables ( $z$ ), and must be formulated to be equal to zero when a complete compliance of the condition is achieved, and very different from zero for clear non-compliance.

As it is explained in reference [11] and [12], the success of optimization approaches requires a correct tuning of a global optimization strategy, and a very careful choice and weighting of all the regularities involved in the process. Because too many regularities can over-constraint the model and too few can lead to an open set of solutions. Nevertheless, some other considerations must also be done. In particular, the departure set of variables is a critical aspect in some senses. Revealing those implications and suggesting some alternative solutions will constitute the core of this paper.

#### 4. INITIAL SOLUTION STRATEGIES

Relevant to the selection of one initial solution for the optimization process, it must be pointed out that the usual strategy up to now is the simplest one of making all  $Z$  coordinates equal to 0 (i.e. the departure 2D graph is used as initial solution). Unfortunately, the departure graph is a local minimum, because many regularities are trivially accomplished on it. For instance, a loop composed by some edges and supposed to correspond to a plane face in the model, always determine a trivial plane face in the graph.

The strategies developed to «escape» from this trivial optimum are all based on heuristic rules to get «tentative» models, and all of them fall into two categories: iterative inflation and direct model generation.

Marill [6] first proposed the inflation strategy, which defined the Objective Function with one single component: the MSDA (Minimum Standard Deviation Angles). The MSDA is not a true regularity, because it does not reflect any properties of the image that must correspond to some properties the searched model also has. It is a heuristic rule, based in the fact that regular polyhedral and convex polyhedral, usually, are very close to accomplish this condition.

In the approach by Leclerc and Fischler [7], the Objective Function was balanced with a varying parameter  $\lambda$  in the following form:  $F = F_A + \lambda F_I + (1-\lambda) F_O$ . Where  $F_I$  (the «inflation» part) includes a set of conditions that are not trivially accomplished in the initial solution ( $z=0$ ), but are not true regularities, and, hence, do not need to be accomplished in the optimum. On the contrary,  $F_O$  (the «optimization» part) contains those true regularities that are trivially accomplished in the initial solution. Finally,  $F_A$  includes all those regularities that can be always applied, because they are not trivially accomplished in the initial solution.

The strategy was defined to allow the optimization algorithm to escape from trivial solution using inflation criteria, while guarantying that the final solution depends only on true regularities. Nevertheless, undesired oscillations can happen, and some intermediate local minimum can be the result of the inflation part. Consequently, alternative initial solutions strategies must be developed, to avoid the use of the «inflation» strategy.

The best alternative seems to be to select as initial solution someone that accomplishes some heuristic rules. In other words, the approach is to define a deterministic process to obtain a «tentative» 3D model whose topology is as close as possible to the final one. This is the case of the «preliminary reconstruction» proposed by Lipson and Sphitalni [10], and based on analyzing the angular distribution graph of lines, to obtain the prevailing angles, and hence the main directions. When three prevailing angles are obtained, the orthogonal perceptual rule («those angles must correspond to three orthogonal directions») is applied to obtain a tentative model.

Another alternative points out to a classification of the objects to be reconstructed according to their characteristic. For instance, in the work by De Bonet [13] the recognition-of-sketches strategy is linked to a previous process where a classification (a «grammar») of the models is carried out based on what he calls the «grammatical structure» within the lines, which yields only a single interpretation.

## 5. OUR RECONSTRUCTION ALGORITHM

In this paper, two different strategies for direct generation from one initial model are presented. According to our experience, they two raise models very close to the psychologically plausible one, but only when applied to some particular kind of models (i.e. to some «grammar» or model classification).

We do establish a simple model classification for polyhedral objects. We do distinguish «normalon» (here we use an extension to 3D to the definition by Dori [14] of a normalon as «a polygon having the property that the angle between any two of its adjacent sides is  $90^\circ$ ), pyramidal and prismatic polyhedral objects.

The first of those strategies is applied to normalon polyhedral models, and is based on the core definition of the orthogonal axonometric projection. The foundation of this approach comes from the very well known dependence between the axonometric coefficients and the angles between each pair of the projected reference axis. In our case, we have reformulated it to obtain the dependence between the angle defined by the projection of every pair of 3D edges connected to the same node and the angle formed by every one of the edges with its own projection segment. Of course, assuming than the corresponding segments in the departure image are oriented parallel to main axis.

The second strategy is based on the relation between proximity and visibility and seems to work properly when applied to convex polyhedral. We do establish an automatic assignment of z coordinates to nodes in a way to guarantee than the visible edges are positioned closer than the non-visible ones. In a second level we can also distinguish among different topological configuration of nodes in the departure image (we do consider W-vertex, Y-vertex, T-vertex, etc.)

Those approaches have been implemented an tested on a program called REFER, that is been developed by the authors [11]. The application is developed using Microsoft's Developer Studio, to run on PC platforms under Windows NT or Windows 95. C++ is used to implement the calculations and data management. Graphical User Interaction is achieved by calling win32 operations through Visual C++. And Open GL was chosen as the best alternative to generate outputs of 3D models.

### 5.1 Axonometric inflation.

The axonometric inflation method is based on the calculation of the angle defined by every edge in a normalon model and its projection in the departure graph, assuming than the graph is an orthogonal axonometry of the model.

It is well know that a relation can be established between the angles defined by the orthogonal projections of three concurrent orthogonal segments (like AD, BD and CD in figure 3) and the angles that every one of those segments determine with its own projection. So, the angle  $\phi$  between one edge CD and its own projection segment C'D' can be stated as follows:

$$\phi = \text{asin} \sqrt{\cotg \alpha \cdot \cotg \beta} \quad (2)$$

where  $\alpha$  and  $\beta$  are, respectively, the angles between  $C'D'$  (the projection of edge CD) and the projections  $A'D'$  and  $B'D'$  of the other two orthogonal edges (AD and BD), when all three are connected to the same node D, as can be seen in figure 3.

Then, the calculation of z coordinates of nodes A, B and C can be easily done. For instance, the relative z coordinate of vertex C can be obtained (see figure 4) as:

$$\text{ABS}(z_C - z_D) = C'D' \cdot \text{atan} \phi \quad (3)$$

where  $C'D'$  is the length of the projection of the edge CD, and  $z_C$  and  $z_D$  are the respective z coordinates of both vertices in the reconstructed edge.

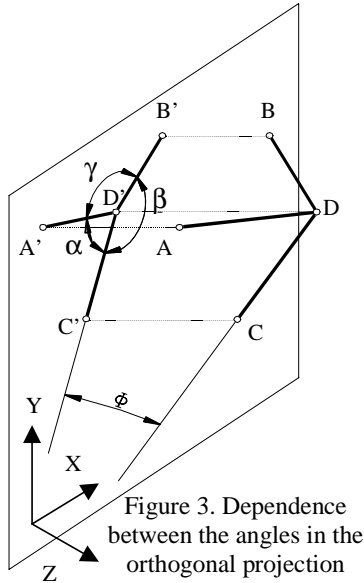


Figure 3. Dependence between the angles in the orthogonal projection

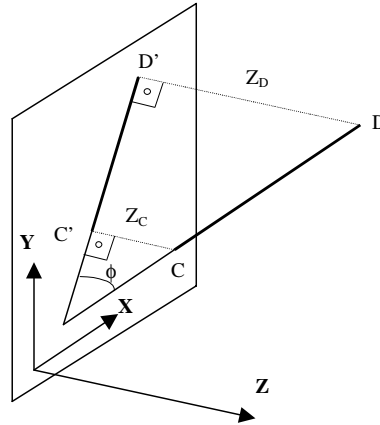


Figure 4. Relation between angle and z coordinates

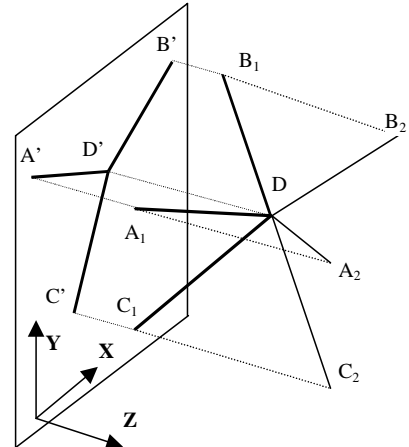


Figure 5. Necker reversion

The coordinates obtained by (3) are relative to the z coordinate of node D. Nevertheless, for reconstruction purposes the z coordinate of vertex D can be arbitrarily assigned without loss of generality, because the trihedron shape will not change, only its position will be different. In addition to this «displacement» degree of freedom, the solution still is not unique, because a reversion of all three angles would lead to a second valid solution. (see figure 5), getting, in this way, the effects of Necker reversion.

Fixing  $z_D = 0$  solves the first problem. Then, to choose one of the two alternatives in the Necker reversion, and, simultaneously, solve computational ambiguities, some considerations have to be taken into account when (3) is implemented. In REFER, the following criterion has been established to determine  $Z_C$ :

- if  $Y_{D'} < Y_{C'}$  then  $Z_C = Z_D - C'D' \tan(\phi)$
- if  $Y_{D'} > Y_{C'}$  then  $Z_C = Z_D + C'D' \tan(\phi)$
- if  $Y_{D'} = Y_{C'}$  (horizontal projection border) then:
  - if  $X_{D'} < X_{C'}$   $Z_C = Z_D - C'D' \tan(\phi)$ .
  - if  $X_{D'} > X_{C'}$   $Z_C = Z_D + C'D' \tan(\phi)$ .

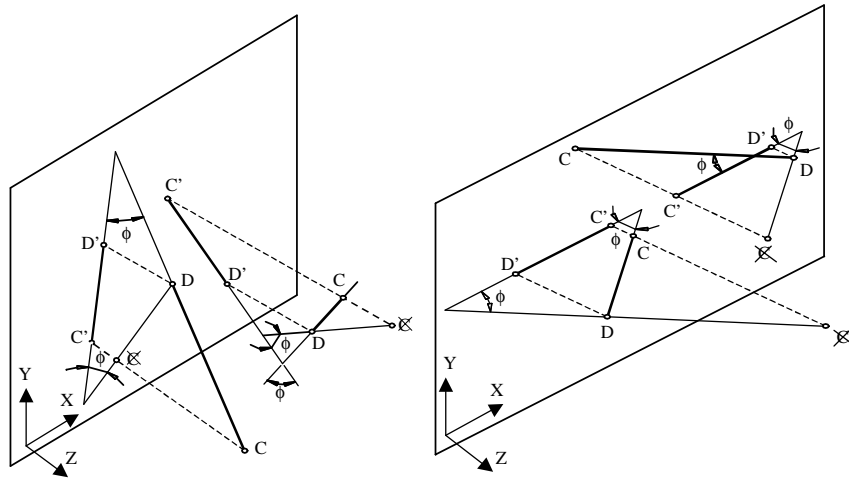


Figure 6. Selection criteria of the angle  $\phi$  position.

The approach can be repeated using any vertex in the graph as a central vertex, and so on until z coordinates all vertices have been determined. It must be noticed that the lack of definition in z coordinates of central vertices can be avoided by simply using as a central vertex any one connected to a previously determined vertex. For instance, by using vertices A, B or C as second central vertex, we are sure to know its Z coordinate, because it has been determined in the previous step, when D was the central vertex.

The restriction to the axonometric-inflation is inferred from its own analytical expression. It is evident that the method makes sense only when the following condition is verified:

$$\cotg \alpha \cdot \cotg \beta \geq 0 \quad (4)$$

This condition determines that the axonometric-inflation will give exact results only when applied to objects with normalon typology projected according to orthogonal and parallel projections. Under such conditions, initial perfect models are obtained, and no one optimization procedures are required to refine the model (as it can be seen in the examples shown in figure 7).

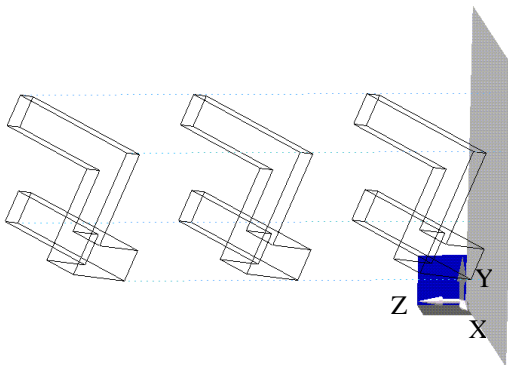


Figure 7. Reconstruction obtained by axonometric inflation

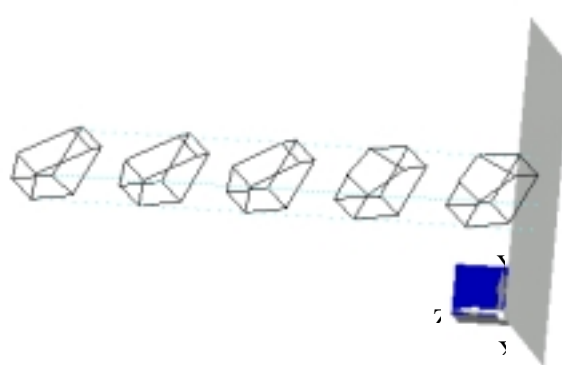


Figure 8. Reconstruction following a preinflation by axonometric-inflation

Nevertheless, when applied to non-normalon polyhedral, or when applied to non orthogonal projections (like cavalier axonometry), the resulting 3D model is useful as a departure model to carry out an optimization process without inflation regularities. Only true regularities must be taken into account to ensure a good solution is obtained by the optimization algorithm (see figure 8).

## 5.2 Levels inflation

The second proposed approach allows preliminary inflation in models of various typologies. It is supposed to be a good alternative to MSDA, because it consists in a direct generation of the model, and it does not require optimization process.

The approach is founded on vertices labeling methods, but only a simple classification of vertices is done (and no one check about correctness of the graph is accomplished). Vertex are classified according to their typology. Different «levels» are assigned to each vertex typology. This means a relative z coordinate is supposed to fit to each vertex typology. Then heuristic z coordinates are assigned to every level, and a tentative model is so obtained.

In REFER four different typologies have been implemented; between level «0» and level «3». The heuristic comes from a direct observation of the axonometric projection of a cube (see figure 9).

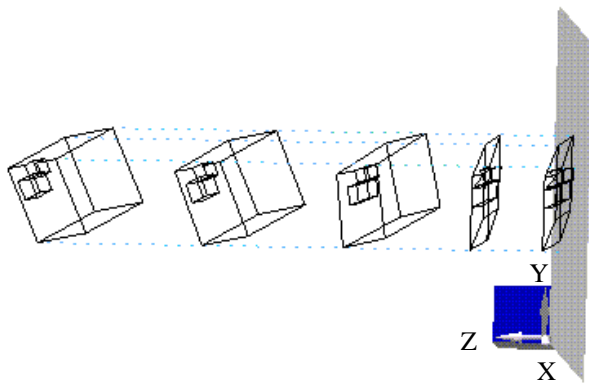


Figure 10. Initial model obtained by levels inflation and optimization

The proposed approach can be improved when hidden lines are specified in the departure graph. Then, a new classification of vertices typology can be done to ensure that partially of fully hidden vertex are assigned lower z coordinates (see figure 11).

As an alternative, hidden lines can be automatically obtained when the departure graph do not consider them. This is accomplished by first executing a face detection algorithm (like the one proposed by Sphitalni and Lipson [15]), Then, a hidden line algorithm detection of our own is used to add hidden lines information to the departure graph.

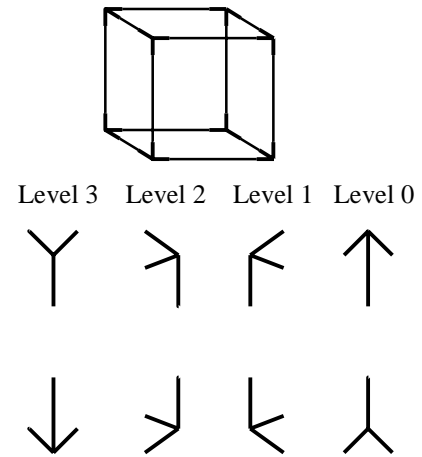


Figure 9. Vertices labeling in a graph

After the identification and joining of the different graph vertices, Z values will be assigned in crescent order from level «0» to level «3».

As a result of the execution of the propose strategy, we obtain initial inflation models like the one which is shown in figure 10 (the model is the closer to the departure plane). The effects of a latter optimization process are also shown in the subsequent chain of tentative models.

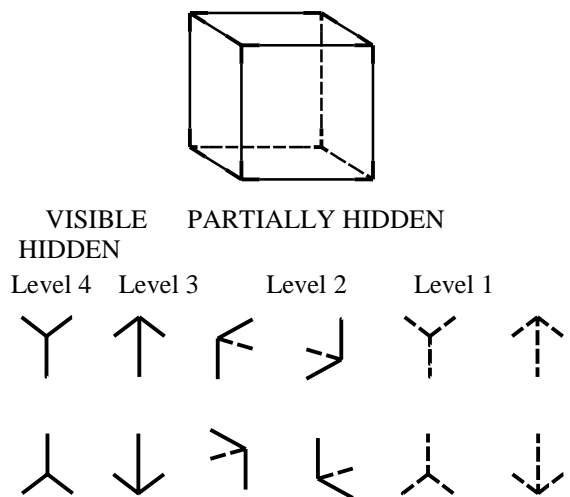


Figure 11. Labelling of the vertices in a graph with hidden borders

A better inflation is obtained when hidden lines are considered. In figure 12 we can see the difference in the initial levels inflation in two cases. In addition, it must be observed that the optimization process can solve both situations to reach the same final topology. Nevertheless, the final model is better and the cost is reduced when hidden lines are used.

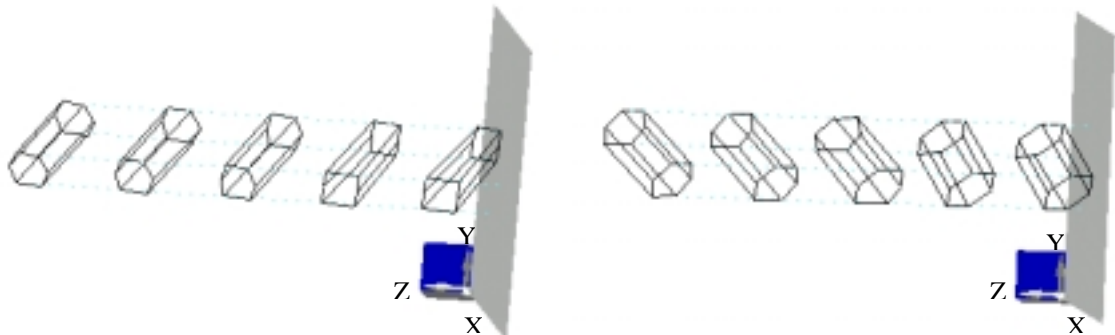


Figure 12. Inflation for levels without hidden lines in contrast to inflation for levels with hidden lines

## 6. CONCLUSIONS

Automatic solid-model generation from engineering drawings is the most efficient way to establish a fluid communication between designers and CAD systems. This is the challenge of 3D reconstruction.

A general-purpose and automatic (or, at least, easy-to-use) system to reconstruct objects is our present objective.

To achieve this goal some «pure computer world» and some «pure Engineering Graphics» aspects must be putted together. In the former, better optimization approaches and a good tuning of the process is required. In the latter, a better definition of 2D regularities and a comprehensive 3D-model classification must be done.

In addition, and due to the great variety of models to be reconstructed, a general approach has proved to be no valid. Consequently, some heuristic and particular strategies seem to be a promising solution. In particular, Initial inflation strategies is one of such approaches, which has proven to be valid when defined according to the typologies of the objects under considered.

To make this approach fully operative, a better and comprehensive classification of models must be done. And some automatic, and, may be, heuristic, rules to choose the appropriate inflation strategy to each typology must be developed.

Nevertheless, the heuristic inflation rules seem to be a good approach to directly solve the simplest models and, simultaneously serve as a departure model to improve efficiency in the regularities based optimization process.

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