VII Appendix IRLE article: Regulating Vertical Relations in the presence of Retailer Differentiation Costs

VII.1 Mathematical Computations in Section IV

VII.1.1 Retail Competition

Retailers' Pricing Stage From equations (8) and (9), we set up the first order conditions for firms A and B:

$$\frac{\partial \Pi_A}{\partial p_A} = \frac{2\,(R-n) - 6p_A + p_B + t(\alpha+\beta)}{2t} = 0, \tag{20}$$

$$\frac{\partial \Pi_B}{\partial p_B} = \frac{2\,(R-n) - 6p_B + p_A + t(\alpha+\beta)}{2t} = 0. \tag{21}$$

From them we obtain the system of best response functions [32]:

$$p_A = \frac{1}{3}\left(R - n + \frac{p_B + t(\alpha+\beta)}{2}\right), \tag{22}$$

$$p_B = \frac{1}{3}\left(R - n + \frac{p_A + t(\alpha+\beta)}{2}\right). \tag{23}$$

Whose solution gives us the equilibrium commercial margins:

$$p_A^* = p_B^* = \frac{1}{5}\left[2\,(R-n) + t\,(\alpha+\beta)\right]. \tag{24}$$

And the corresponding equilibrium quantities:

$$q_A^* = q_B^* = \frac{3}{10t}\left[2\,(R-n) + t\,(\alpha+\beta)\right]. \tag{25}$$

In equilibrium, the indifferent consumer's location will be at a distance from A:

$$x_o^* = \frac{\alpha+\beta}{2}, \tag{26}$$

[32] We check the second order conditions: $\frac{\partial^2 \Pi_A}{\partial p_A^2} = -\frac{3}{t} < 0$, and $\frac{\partial^2 \Pi_B}{\partial p_B^2} = -\frac{3}{t} < 0$.

and:

$$x_A^* = x_B^* = \frac{3\,(R-n) - t\,(\alpha+\beta)}{5t}. \tag{27}$$

Equilibrium profits of the two retailers are then:

$$\Pi_A^* = \frac{3\left[2\,(R-n) + t\,(\alpha+\beta)\right]^2}{50t} - c\alpha^2 - F, \tag{28}$$

$$\Pi_B^* = \frac{3\left[2\,(R-n) + t\,(\alpha+\beta)\right]^2}{50t} - c\beta^2 - F. \tag{29}$$

Retailers' Location Stage We obtain the derivatives of the profit functions with respect to the corresponding location variables α and β and we set up the F.O.C. of the location stage:

$$\frac{\partial \Pi_A}{\partial \alpha} = \frac{3\left[2\,(R-n) + t\,(\alpha+\beta)\right]}{25} - 2c\alpha = 0, \tag{30}$$

$$\frac{\partial \Pi_B}{\partial \beta} = \frac{3\left[2\,(R-n) + t\,(\alpha+\beta)\right]}{25} - 2c\beta = 0. \tag{31}$$

Which give us the following system of best response functions:

$$\alpha = 3\frac{2R - 2n + t\beta}{50c - 3t}, \qquad \beta = 3\frac{2R - 2n + t\alpha}{50c - 3t}. \tag{32}$$

Whose solution gives us the equilibrium locations of the retailers[33].

$$\alpha^* = \beta^* = \frac{3\,(R-n)}{25c - 3t}. \tag{33}$$

The equilibrium locations of A and B require $c > \frac{3t}{25}$ in order for α and β to be positive. It is very difficult that retail competition produces *minimum differentiation* ($\alpha^* = \beta^* = 0$), in fact, it requires that $c \to \infty$. Furthermore, given our assumption $c \geq t$, *maximum differentiation* ($\alpha^* = \beta^* = +\infty$) is impossible (only if $c = \frac{3t}{25}$).

[33] Let's check that the S.O.C. hold: $\frac{\partial^2 \Pi_A}{\partial \alpha^2} = \frac{3}{25}t - 2c < 0$, and $\frac{\partial^2 \Pi_B}{\partial \beta^2} = \frac{3}{25}t - 2c < 0$, require $c > \frac{3t}{50}$.

Therefore, we find a continuum of interior equilibria depending on the relationship between the differentiation cost parameter and the transportation cost parameter. The greater the differentiation cost parameter, as compared to the transportation cost parameter, the less differentiation we have in equilibrium.

With the values obtained for α^* and β^* we can substitute in the functions (24), (25), (28), and (29) calculated before:

$$p_A^* = p_B^* = \frac{10c(R-n)}{25c-3t}, \tag{34}$$

$$q_A^* = q_B^* = \frac{15c\,(R-n)}{(25c-3t)\,t}, \tag{35}$$

$$\Pi_A^* = \Pi_B^* = \frac{3c\,(R-n)^2\,(50c-3t)}{(25c-3t)^2\,t} - F. \tag{36}$$

Manufacturer's Pricing Stage Now we can solve for M's optimisation problem concerning n and F, which is described as:

$$\begin{aligned} \max_{n,F} \quad \Pi_M &= n\cdot(q_A^*+q_B^*)+2F \\ s.t. \quad \Pi_A^* &\geq 0\,,\ \Pi_B^* \geq 0. \end{aligned} \tag{37}$$

The manufacturer will choose $F = \frac{3c(R-n)^2(50c-3t)}{(25c-3t)^2 t}$, so that $\Pi_A^* = \Pi_B^* = 0$. And then he will maximise with respect to the standard product intermediate price n the resulting function:

$$\Pi_M = \frac{6c\,(R-n)\,(50Rc+75nc-3tR-12tn)}{(25c-3t)^2\,t}. \tag{38}$$

The optimal n will satisfy:

$$\frac{\partial \Pi_M}{\partial n} = \frac{6c\,(25Rc-150nc-9tR+24tn)}{(25c-3t)^2\,t} = 0, \tag{39}$$

whose solution gives[34]:

[34] The S.O.C. $\frac{\partial^2 \Pi_M}{\partial n^2} = 36c\frac{-25c+4t}{(3t-25c)^2 t} < 0$ holds if $c > \frac{4t}{25}$.

$$n^* = \frac{R(25c - 9t)}{6(25c - 4t)}. \quad (40)$$

VII.1.2 Resale Price Maintenance

Manufacturer's Retail Price Decision The manufacturer can obtain, thanks to the fixed payment F, all the profits of the retailers. Besides, instead of charging the intermediate price n, the manufacturer will, under the RPM distribution mode, choose the retail price.

$$\begin{aligned} \max_{p_A, p_B} \quad \Pi_M &= F_A + F_B \\ s.t. \quad \Pi_A^* &\geq 0\,,\ \Pi_B^* \geq 0. \end{aligned} \quad (41)$$

The manufacturer will choose $F_A = p_A \left(\frac{2R - 3p_A + p_B + t(\alpha+\beta)}{2t}\right) - c\alpha^2$, and $F_B = p_B \left(\frac{2R - 3p_B + p_A + t(\alpha+\beta)}{2t}\right) - c\beta^2$, so that $\Pi_A^* = \Pi_B^* = 0$. So, now he will maximise with respect to the retail prices his profit function[35].

$$\frac{\partial \Pi_M}{\partial p_A} = \frac{2(R - 3p_A + p_B) + t(\alpha + \beta)}{2t} = 0, \quad (42)$$

$$\frac{\partial \Pi_M}{\partial p_B} = \frac{2(p_A + R - 3p_B) + t(\alpha + \beta)}{2t} = 0. \quad (43)$$

From them we obtain the best response functions [36]:

$$p_A = \frac{1}{6}\left[2(R + p_B) + t(\alpha + \beta)\right], \qquad p_B = \frac{1}{6}\left[2(R + p_A) + t(\alpha + \beta)\right]. \quad (44)$$

Then we obtain the equilibrium prices:

$$p_A^* = \frac{1}{4}\left[2R + t(\alpha + \beta)\right], \qquad p_B^* = \frac{1}{4}\left[2R + t(\alpha + \beta)\right]. \quad (45)$$

The equilibrium profits of A and B as function of α and β are:

[35] See equations (11) and (12).

[36] We check the second order conditions: $\frac{\partial^2 \Pi_M}{\partial p_A^2} = -\frac{3}{t} < 0$,and $\frac{\partial^2 \Pi_M}{\partial p_B^2} = -\frac{3}{t} < 0$.

$$\Pi_A^* = \frac{\left[2R + t(\alpha + \beta)\right]^2}{16t} - c\alpha^2 - F, \qquad \Pi_B^* = \frac{\left[2R + t(\alpha + \beta)\right]^2}{16t} - c\beta^2 - F. \quad (46)$$

Retailers' Location Stage The retailers will independently make the location decision under the distribution mode of RPM:

$$\frac{\partial \Pi_A}{\partial \alpha} = \frac{1}{8}\left[2R + t(\alpha + \beta)\right] - 2c\alpha = 0, \quad (47)$$

$$\frac{\partial \Pi_B}{\partial \beta} = \frac{1}{8}\left[2R + t(\alpha + \beta)\right] - 2c\beta = 0. \quad (48)$$

From the F.O.C. we obtain the best response functions:

$$\alpha = \frac{2R + t\beta}{16c - t}, \qquad \beta = \frac{2R + t\alpha}{16c - t}. \quad (49)$$

And the equilibrium locations of the retailers are[37]:

$$\alpha^* = \frac{R}{8c - t}, \qquad \beta^* = \frac{R}{8c - t}. \quad (50)$$

In the case of Resale Price Maintenance, as in that of Retail Competition, it is very difficult to observe the extreme differentiation results (we will have minimum differentiation only if $c \to \infty$, and maximum differentiation only if $c = \frac{t}{8}$, which is not possible under our assumptions). The equilibrium locations of the retailers are quite close in both distribution modes, only in RPM the retailers will incur in a little more differentiation than under COMP.

VII.1.3 Image Maintenance

The profit functions of COMP depending on α and β were (28) and (29). And, the profit function that the manufacturer will maximise is:

[37]The S.O.C. hold if we consider our assumption that $c \geq t$, $\frac{\partial^2 \Pi_A}{\partial \alpha^2} = \frac{1}{8}t - 2c < 0$ and $\frac{\partial^2 \Pi_B}{\partial \beta^2} = \frac{1}{8}t - 2c < 0$.

$$\max_{\alpha,\beta} \quad \Pi_M = n \cdot (q_A^* + q_B^*) + F_A + F_B \tag{51}$$
$$s.t. \quad \Pi_A^* \geq 0\,,\ \Pi_B^* \geq 0.$$

The manufacturer will choose his franchise fee so that $\Pi_A^* = \Pi_B^* = 0$, so $F_A = \frac{3[2(R-n)+t(\alpha+\beta)]^2}{50t} - c\alpha^2$, and $F_B = \frac{3[2(R-n)+t(\alpha+\beta)]^2}{50t} - c\beta^2$. And then he will maximise with respect to the retailers' locations α and β his profit function:

$$\Pi_M = \frac{3\left[2\left(R-n\right)+t\left(\alpha+\beta\right)\right]^2 - 25ct\left(\alpha^2+\beta^2\right) + 15n\left[2\left(R-n\right)+t\left(\alpha+\beta\right)\right]}{25t}. \tag{52}$$

Manufacturer's Location Decision The F.O.C. of the manufacturer's problem are:

$$\frac{\partial \Pi_M}{\partial \alpha} = \frac{3}{25}\left[4R+n+2t\left(\alpha+\beta\right)\right] - 2c\alpha = 0, \tag{53}$$

$$\frac{\partial \Pi_M}{\partial \beta} = \frac{3}{25}\left(4R+n+2t\left(\alpha+\beta\right)\right) - 2c\beta = 0. \tag{54}$$

From them we get the best response functions:

$$\alpha = \frac{3\left(4R+n+2t\beta\right)}{2\left(25c-3t\right)}, \qquad \beta = \frac{3\left(4R+n+2t\alpha\right)}{2\left(25c-3t\right)}. \tag{55}$$

And the equilibrium locations under the IM mode of distribution[38]:

$$\alpha^* = \frac{3\left(4R+n\right)}{2\left(25c-6t\right)}, \qquad \beta^* = \frac{3\left(4R+n\right)}{2\left(25c-6t\right)}. \tag{56}$$

Manufacturer's Pricing Decision The manufacturer also chooses the price of the standard product in this regime in order to maximise his profits:

$$\Pi_M = \frac{3\left(4Rnc+3n^2t+8R^2c-12n^2c\right)}{2t\left(25c-6t\right)}. \tag{57}$$

[38]The S.O.C. hold under our assumptions: $\frac{\partial^2 \Pi_M}{\partial \alpha^2} = \frac{6}{25}t - 2c < 0$, and $\frac{\partial^2 \Pi_M}{\partial \beta^2} = \frac{6}{25}t - 2c < 0$.

The F.O.C. is:

$$\frac{\partial \Pi_M}{\partial n} = \frac{3\,(2Rc + 3nt - 12nc)}{t\,(25c - 6t)} = 0. \tag{58}$$

And the optimal intermediate price[39]:

$$n^* = \frac{2cR}{12c - 3t}. \tag{59}$$

VII.1.4 Complete Control

The profit functions of RPM depending on α and β were those in equation (46). Let us remind that in CC and in RPM, p_A and p_B are final retail prices, not profit margins, so the manufacturer will not charge any intermediate price ($n = 0$). The manufacturer will optimise with respect to locations:

$$\begin{aligned} \max_{\alpha,\beta} \quad \Pi_M &= F_A + F_B \\ s.t. \quad \Pi_A^* &\geq 0\,,\ \Pi_B^* \geq 0. \end{aligned} \tag{60}$$

The manufacturer will choose $F_A = \frac{[2R+t(\alpha+\beta)]^2}{16t} - c\alpha^2$, and $F_B = \frac{[2R+t(\alpha+\beta)]^2}{16t} - c\beta^2$, so that $\Pi_A^* = \Pi_B^* = 0$. And then he will maximise with respect to the retailers' locations α and β his profit function:

$$\Pi_M = \frac{[2R + t\,(\alpha+\beta)]^2}{8t} - c\left(\alpha^2 + \beta^2\right). \tag{61}$$

Manufacturer's Location Decision The F.O.C. of the manufacturer's location choice for the retailers are:

$$\frac{\partial \Pi_M}{\partial \alpha} = \frac{1}{2}R + \frac{1}{4}t\,(\alpha+\beta) - 2c\alpha = 0, \tag{62}$$

$$\frac{\partial \Pi_M}{\partial \beta} = \frac{1}{2}R + \frac{1}{4}t\,(\alpha+\beta) - 2c\beta = 0. \tag{63}$$

[39] The S.O.C. holds: $\frac{\partial^2 \Pi_M}{\partial n^2} = -9\frac{4c-t}{t(25c-6t)} < 0$.

And the best response functions:

$$\alpha = \frac{2R + t\beta}{8c - t}, \qquad \beta = \frac{2R + t\alpha}{8c - t}. \tag{64}$$

So, the manufacturer's profit maximising locations are[40]:

$$\alpha^* = \frac{R}{4c - t}, \qquad \beta^* = \frac{R}{4c - t}. \tag{65}$$

Note that these location equations are identical to those in IM. The locations under IM and CC produce more differentiation in equilibrium than RPM or COMP.

VII.1.5 Exclusive Territories

Retailer's Pricing Decision When choosing his retail price the retailer is also determining his captive market, and in the ET distribution mode, this is equivalent to his location.

The first order condition of the maximisation problem is:

$$\frac{\partial \Pi_A}{\partial p_A} = \frac{2\left[R\left(t + c\right) - p_A\left(2t + c\right)\right]}{t^2} = 0.$$

From where we extract the maximising[41] price for the retailer:

$$p_A^* = p_B^* = \frac{R\left(c + t\right)}{c + 2t}. \tag{66}$$

With these prices, the locations will be:

$$\alpha^* = \beta^* = \frac{R}{c + 2t}. \tag{67}$$

This time both differentiation and transportation costs play against differentiation. But the degree of differentiation attained under ET is greater than for any of the other distribution modes considered until now. Only for $c = t$ it will coincide with IM and CC.

[40]The S.O.C. hold: $\frac{\partial^2 \Pi_M}{\partial \alpha^2} = \frac{1}{8}\frac{2t^2 - 16ct}{t} < 0$ and $\frac{\partial^2 \Pi_M}{\partial \beta^2} = \frac{1}{8}\frac{2t^2 - 16ct}{t} < 0$.

[41]We check that the second order condition holds: $\frac{\partial^2 \Pi_A}{\partial p_A^2} = -2\frac{2t + c}{t^2} < 0$.

VII.1.6 Retail Monopoly

Retailer's Pricing Decision The monopolist retailer only has to decide his retail price, because his location will obviously be zero, given that consumer density is the same along the line and the retailer does not need to incur in any differentiation cost to reduce competition. We set up the F.O.C. of the maximisation problem:

$$\frac{\partial \Pi_A}{\partial p_A} = \frac{2\left(R - p_A\right)}{t} = 0. \tag{68}$$

The maximising price is[42]:

$$p_A^* = \frac{R}{2}. \tag{69}$$

VII.2 Proof of Proposition 2

Let us compare the equilibrium retailers' locations under every distribution mode:

$\alpha_{ET} - \alpha_{IM=CC} = \frac{R}{2t+c} - \frac{R}{4c-t} = \frac{3R(c-t)}{(2t+c)(4c-t)}$. This difference is always positive, remember our assumption $c \geq t$, it can only be zero when $c = t$.

$\alpha_{IM=CC} - \alpha_{RPM} = \frac{R}{4c-t} - \frac{R}{8c-t} = \frac{4cR}{(4c-t)(8c-t)}$. This difference is always positive.

$\alpha_{RPM} - \alpha_{COMP} = \frac{R}{8c-t} - \frac{5R}{2(25c-4t)} = \frac{R(10c-3t)}{2(8c-t)(25c-4t)}$. This difference is always positive.

Besides, as $c \to \infty$ the denominators of all the location expressions go to infinity, so location goes to zero. Furthermore, this happens gradually, as c grows in terms of t. **QED**

VII.3 Proof of Proposition 3

Let us compare the manufacturer's equilibrium profit under every distribution mode:

$\Pi_{IM=CC} - \Pi_{ET} = \frac{2cR^2}{t(4c-t)} - \frac{2R^2}{2t+c} = \frac{2R^2\left(c^2-2ct+t^2\right)}{t(4c-t)(2t+c)}$. This difference is always positive, remember our assumption $c \geq t$, it can only be zero when $c = t$.

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[42] Let us check that the S.O.C. holds $\frac{\partial^2 \Pi_A}{\partial p_A^2} = -\frac{4}{t} < 0$.

$\Pi_{IM=CC} - \Pi_{RPM} = \frac{2cR^2}{t(4c-t)} - \frac{2cR^2(16c-t)}{t(8c-t)^2} = \frac{8R^2c^2}{(4c-t)(8c-t)^2}$. This difference is always positive, so $\Pi_{IM=CC} > \Pi_{RPM}$ $\forall c$.

If $c < 1.511 \cdot t$ we have that $\Pi_{ET} > \Pi_{RPM}$, while if $c > 1.511 \cdot t$ then $\Pi_{ET} < \Pi_{RPM}$.

$\Pi_{RPM} - \Pi_{COMP} = \frac{2cR^2(16c-t)}{t(8c-t)^2} - \frac{25cR^2}{2t(25c-4t)} = \frac{cR^2(44c-9t)}{2(8c-t)^2(25c-4t)}$. This difference is always positive, so $\Pi_{RPM} > \Pi_{COMP}$ $\forall c$.

If $c < 1.6 \cdot t$ we have that $\Pi_{ET} > \Pi_{COMP}$, while if $c > 1.6 \cdot t$ then $\Pi_{ET} < \Pi_{COMP}$.

$\Pi_{COMP} - \Pi_{MON} = \frac{25cR^2}{2t(25c-4t)} - \frac{R^2}{2t} = \frac{2R^2}{25c-4t}$.This difference is always positive, so $\Pi_{COMP} > \Pi_{MON}$ $\forall c$.

If $c < 2 \cdot t$ we have that $\Pi_{ET} > \Pi_{MON}$, while if $c > 2 \cdot t$ then $\Pi_{ET} < \Pi_{MON}$.

Finally, we can see, by calculating the limits, that Π_{COMP}, $\Pi_{CC} = \Pi_{IM}$, and $\Pi_{RPM} \longrightarrow \Pi_{MON} = \frac{R^2}{2t}$ when $c \longrightarrow \infty$. **QED**

VII.4 Proof of Proposition 4

Let us compare social welfare under every distribution mode:

$$W_{COMP} - W_{IM=CC} = \frac{25R^2\left(75c^2-8tc-2t^2\right)}{4t(25c-4t)^2} - \frac{2R^2\left(6c^2-tc-t^2\right)}{t(4c-t)^2} =$$
$$= \frac{3R^2\left(1769c^2t-1200c^3-424t^2c+26t^3\right)}{4(-25c+4t)^2(4c-t)^2}.$$

This difference is positive if $c < 1.1932803 \cdot t$ and it is negative if $c > 1.1932803 \cdot t$.

$$W_{COMP} - W_{RPM} = \frac{25R^2\left(75c^2-8tc-2t^2\right)}{4t(25c-4t)^2} - \frac{2R^2\left(24c^2-tc-t^2\right)}{t(8c-t)^2} =$$
$$= \frac{R^2\left(2203c^2t+600c^3-872t^2c+78t^3\right)}{4(-25c+4t)^2(8c-t)^2}.$$

This difference is always positive, remember our assumption $c \geq t$, so $W_{COMP} > W_{RPM}$ $\forall c$.

If $c < 1.049 \cdot t$ we have that $W_{RPM} > W_{CC} = W_{IM}$, while if $c > 1.049 \cdot t$ we have: $W_{RPM} < W_{CC} = W_{IM}$.

$W_{RPM} - W_{ET} = \frac{2R^2\left(24c^2-tc-t^2\right)}{t(8c-t)^2} - \frac{2R^2(3t+c)}{(2t+c)^2} = \frac{2R^2\left(31c^3t+24c^4+39t^3c-7t^4-85c^2t^2\right)}{t(8c-t)^2(2t+c)^2}$. This difference is always positive, so $W_{RPM} > W_{ET}$ $\forall c$.

$W_{RPM} - W_{MON} = \frac{2R^2\left(24c^2-tc-t^2\right)}{t(8c-t)^2} - \frac{3R^2}{4t} = \frac{R^2(40c-11t)}{4(8c-t)^2}$. This difference is always positive so $W_{RPM} > W_{MON}$ $\forall c$.

If $c = t$ we have that $W_{CC} = W_{IM} = W_{ET}$, while if $c > t$ we have: $W_{CC} = W_{IM} > W_{ET}$.

If $c < 1.44 \cdot t$ we have that $W_{ET} > W_{MON}$, while if $c > 1.44 \cdot t$ we have: $W_{ET} < W_{MON}$.

Finally, we can see, by calculating the limits, that W_{COMP}, $W_{CC} = W_{IM}$, and $W_{RPM} \longrightarrow W_{MON} = \frac{3R^2}{4t}$ when $c \longrightarrow \infty$. **QED**